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9 Mathematics beyond and across the curriculum

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Introduction

Mathematics has a privileged status in education as a subject that is taught universally and to all ages in schools. This status partly reflects the practical utility of the tools and concepts of mathematics, as they are applied to everything from school tasks found in mathematics textbooks to realistic situations like daily business transactions and the statistics of the latest news story. There is also an assumption that participation in school mathematics is the best way for pupils and students to learn how to think abstractly (Schoenfeld, 2017).

The topic of crosscurricular teaching has a long history and has also become an issue within the community of researchers in mathematics education (e.g., Doig et al., 2019; Ward-Penny, 2011). The advocates of crosscurricular teaching and learning speak of the advantages in helping students develop a deeper understanding of concepts through authentic activities and help them encounter the *Big Ideas* of mathematics (Charles, 2005; Toh & Yeo, 2019; Ward-Penny, 2011). While helping students forge meaningful connections across ideas that are central to the learning of mathematics, a pivotal goal is to also make the curriculum more relevant and motivating to students (Czerniak & Johnson, 2014; Ward-Penny, 2011). However, moving from educational ideas as expressed in the curricula toward the successful implementation of those same ideas in the classroom is not without its challenges. Meier et al. (1998) list barriers to the successful integration of any given set of school subjects, ranging from the lack of common assessment guidelines to rigid teacher beliefs.

In this chapter, we will argue that there are also challenges unique to how mathematics is taught and learned within a cross- and transcurricular setting. These challenges are linked to how mathematics is viewed and perceived as a school subject. It is common to view mathematics as having a strict hierarchical structure and to perceive learning in mathematics as acquiring a set of techniques that can be applied outside of mathematics. We will suggest that this view, which we call *instrumental*, makes mathematics difficult to integrate with other subjects. That this indeed is the case is exemplified with an historical overview of the Finnish curriculum where the friction between viewpoints

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is made explicit. Finally, it is argued that another view on mathematics teaching and learning, which we call *relational*, allows us to see the value of integration differently.

Mathematics from an educational viewpoint

From an educational viewpoint, mathematics can be seen both as a collection of facts, rules, and procedures to be learnt and as a science of patterns and systems in which we investigate problems and hypotheses, reason and discuss, specialize and generalize, conjecture and convince – in short, as a science where we develop abstract mathematical thinking. The word abstract here has a twofold meaning. The first is connected to how a real situation is simplified to construct an abstract mathematical model that contains only the essential features of a problem or situation (see, e.g., Cheng, 2019). The second meaning is closer to how a working mathematician thinks about the power of abstraction. Here we leave the close connections with the physical world and enter the realm of mathematical abstractions, where surprising and deep connections between seemingly different areas of mathematics may emerge. A historical example of this is how the theory of complex numbers unified what was previously thought of as unrelated areas of mathematics, spawning a rich set of new insights and applications. From research we know much about why mathematical thinking is important for both students and teachers, what it takes to learn mathematical thinking, and how to build thinking classrooms (e.g., Liljedahl, 2021; Schoenfeld, 2017). Besides knowing mathematical facts, rules, and procedures as well as when and how to apply these when solving traditional types of mathematical tasks, the students and teachers on all levels of schooling also need to know how to approach and develop more cognitively demanding and inquiry-based mathematical problems or applications (Liljedahl, 2021). This broadens the perspective on mathematics education and shows how mathematics contributes to the Bildung of the student (see Chapter 3).

In the following, we bring up issues that further illuminate mathematics and its learning and help us discuss the role of mathematics in crosscurricular and transcurricular educational settings. We will distinguish between, on the one hand, an instrumental view on mathematics and its learning, and, on the other hand, a relational view. The distinction is remotely related to the one discussed by Skemp (1978) but does not coincide with it. Skemp's focus is on the concept of understanding, while ours is a distinction between two ways of approaching mathematics from an educational viewpoint. Our take on the instrumental view does not exclude a relational understanding, as we regard the two views as complementary. Furthermore, the relational view described here goes beyond Skemp's relational understanding since it includes the relational *qua* social dimensions of learning mathematics.

When seen from the instrumental perspective, the value of mathematics and learning mathematics is taken to lie in the applications to which mathematical facts, rules, and procedures can successfully be put by the user. This view may reveal itself in statements indicating that students know a rule or procedure and can use it for approaching mathematical challenges appearing in the mathematics curriculum or in the curricula of other school subjects. From this perspective, learning mathematics becomes a matter of acquiring a set of techniques, rules, propositions, and the ability to apply them in different situations.

The instrumental view can be seen to join hands with the emphasis on the importance, usefulness, and intrinsic value of possessing mathematical competences that has become commonplace in wake of the Danish KOM project (Niss & Jensen, 2002). A common question asked in this discussion is whether there is a mathematical competence or a plethora of competences that a student needs before entering crosscurricular and transcurricular settings in and outside school. The value of mathematics and learning mathematics is here taken to lie mainly in mathematical sub-competences of a cognitive nature that are considered pertinent for someone in school, everyday life, society, and the labor market. Mathematical competence is broader than knowing how to apply a set of mathematical methods. Following Niss and Højgaard, a student's mathematical competence is constituted by his or her insightful readiness to act, meaning the student acts appropriately in response to all kinds of mathematical challenges pertaining to given situations; situations that need not be mathematical in and of themselves, as long as they (may) generate mathematical challenges (Niss & Højgaard, 2019, p. 12; see also similar notions of "action competence" in Chapters 4 and 13). According to Niss and Højgaard, to act appropriately involves being able to pose and answer questions within and by means of mathematics as well as the ability to handle the language, constructs, and tools of mathematics (Niss & Højgaard, 2019). What remains in the background of, or is totally excluded from the discussion on mathematical competences, are the dispositions of the students, including the capacity of being critical toward the impact of mathematics in society (see Chapter 4). Other aspects that remain in the background are students' emotions, attitudes, and volitionality, as well as the reflexivity between students' school mathematical identity work – including the development of dispositions – and school mathematical traditions (Cobb et al., 2009; Skovsmose & Valero, 2001; Röj-Lindberg, 2017).

Adopting a relational view on mathematics means accounting for the intra-mathematical relations and mathematical competences as well as for the relations between people within and outside school and between these people and mathematics. Besides teachers and students, these people can include parents, peers, etc. By intra-mathematical relations, we intend to convey the intricate connections that exist between mathematical ideas and domains, for example, between arithmetic and algebra. These intra-mathematical relations may appear already on a very elementary level of mathematics studies within such school mathematical traditions that are not governed by restrictive assumptions about what students are capable of learning and in which order. For instance, research has shown that algebraic thinking and the use of algebraic tools is possible as early as in the first grades and beneficial to both the learning of arithmetic and the learning of algebra (e.g., Schliemann et al., 2006). Another aspect, highly important within crosscurricular approaches, is how the relation between informal and formal mathematical languages in use, that is, between discourses, is understood. Within a relational view on mathematics, it is understood as the expansion of repertoires of ways of talking about problems and phenomena. A relation is constructed between less and more formal ways of expressing one's mathematical thinking. Less formal and more formal discourses are not in opposition but work together and in relation to other forms of discourse, including languages in use in other subjects, school discourses, home discourses, and so on (Barwell, 2016). By zooming out from the intra-mathematical relations to the social dimensions of mathematical activity, the relational view allows us to account for learning mathematics as participation as well as to see meaning, thinking, and reasoning as products of social activity (e.g., Lerman, 2000). Thus, the relational view leads up to a Bildung perspective on mathematics education, by also incorporating a focus on the interrelations between the student and the environment (see the discussion in Chapter 3).

Different views on mathematics have direct implications on the role mathematics is assigned in crosscurricular settings. If one views mathematics first and foremost instrumentally, the role of mathematics is easily reduced to one of providing the quantitative toolbox for taking part in a cross-disciplinary project or theme. From a relational perspective, learning mathematics within crosscurricular settings emerges not only in the applications of mathematical facts, rules, and procedures, but also in the sense-making processes where various forms of discourse become treated as mathematical by the participants. For example, a newspaper article might be discussed as an informative text in literature education and become the starting point for inquiries into how mathematical facts are represented and used in the local society. From this perspective, mathematics is learned both *for* and *through* taking part in crosscurricular settings.

Mathematics within crosscurricular settings: cases and problems

While there are many examples of crosscurricular projects involving subjects other than mathematics or where mathematics is hardly visible (see examples in, e.g., McPhail, 2018; Rowley & Cooper, 2009), the examples where mathematics is integrated are not as readily found in the literature. The most common kind of example consists in the integration of the so-called STEM subjects, that is, science, technology, engineering, and mathematics.

Among the expected positive outcomes of STEM integration are increased student motivation and a flexible mindset. Crosscurricular teaching is also expected to prepare the students for grappling with grand societal challenges, sometimes called wicked problems. These assumed positive outcomes and other putative benefits and goals of crosscurricular work are summarized in Chapter 3. In the following, we will refer to some international examples where these positive outcomes are visible. At the same time, these examples indicate what aspects of mathematical practices need to be considered within crosscurricular settings.

Tytler et al. (2019) report on positive effects of crosscurricular STEM projects involving students aged 12–15 in Australia. These projects were based on large-scale initiatives each involving several schools. In three cases studied more closely, the most clearly perceived benefit of crosscurricular work was that student engagement improved. In one of the cases, "the usefulness of mathematics became more evident and [the students] were able to 'transfer' knowledge more readily between their STEM subjects" (Tytler et al., 2019, p. 65). During the initiatives, the teachers' attitudes to the crosscurricular STEM projects changed only gradually in a more positive direction. Tytler and his colleagues associate this shift in attitude with changes in the teachers' pedagogy and to the increased student engagement that followed their work on real-life problems. The authors conclude that the success of STEM integration depends on the use of open-ended tasks that allow for problem-solving and the creative use of mathematics in understanding the problems. The authors expressly advocate against using previously known mathematics as a tool unless this can provide important insights into the problems.

A conclusion drawn from the literature review by Honey et al. (2014) is, likewise, that integration of mathematics and science can be fruitfully furthered if the students are involved in the mathematical modeling process of the natural systems studied. Like Tytler and his colleagues (2019), Honey, Pearson, and Schweingruber suggest that the positive effects of curriculum integration can be more clearly discerned in the students' increased motivation and interest than in outcomes on standardized achievement tests. In a similar vein, Ward-Penny (2011, p. 6) argues that "[c]arefully constructed problem situations might even motivate the learner further, by giving them room to devise their own strategies, carry out their own methods and develop a genuine sense of ownership regarding their work." He warns that a compartmentalized curriculum makes the students search for solutions to problems too narrowly among mathematical skills and competences that are typically learnt during mathematics lessons.

However, as Doig and Jobling (2019) point out, it remains to be seen whether these motivational factors also have positive effects on students' conceptual understanding. In a study from the Netherlands, where students took part in a STEM course in upper secondary school, some students complained that only low-level mathematics was required and that they did not use mathematics skills learnt in the mathematics classroom (den Braber et al., 2019).

If we consider the aforementioned cases from the perspective of the distinction between relational and instrumental views on mathematics education, it seems that the success of crosscurricular learning can be hampered by a one-sided instrumental view. Such a view can even prevent the participants from discerning positive aspects of crosscurricular work. If the contribution of mathematics is taken to consist in a set of quantitative tools that are learnt beforehand and then applied in a crosscurricular setting, there is little room for expanding the toolbox in crosscurricular learning. Mathematics is reduced to a handmaiden to the other subjects and, moreover, one that must be learnt separately. From a relational view, student motivation and engagement are not seen as external to learning mathematics. Furthermore, we argue that the relational view encompasses precisely what Ward-Penny stresses in the aforementioned quote: that students are allowed "to devise their own strategies, carry out their own methods and develop a genuine sense of ownership regarding their work."

Williams and Roth (2019) maintain that the value of crosscurricular approaches that include mathematics lies partly in that mathematics provides necessary tools for quantitative problem-solving and partly in that the crosscurricular setting provides mathematics teaching with a rich context – "the added value of a wider world." In addition, they also stress that students should become aware of the nature of different disciplines and school subjects. A value of crosscurricular projects lies in the fact that they give insights into when a certain subject can add something and when it cannot.

These examples, thus, contain possible ingredients for fruitful crosscurricular teaching. It is worth noting, however, that the cases discussed by Tytler et al. (2019) were part of two large-scale initiatives to further STEM integration. The teachers involved received intensive support from collaborating universities and other stakeholders. Moreover, even with this level of support for subject integration, "a large portion of the mathematics curriculum" was taught independent of the STEM projects in order to meet the requirements of the syllabus. Similar observations concerning the need for external support for the teachers have been noticed by others (e.g., Röj-Lindberg et al., 2022).

Regarding the *assessment* of crosscurricular teaching, there are problems facing researchers and teachers. Honey et al. (2014) point out that if the instruments for measuring learning are devised within a subject-based setting, they will fail to detect at least some of the benefits of the crosscurricular activity. Another problem concerns the outcome of crosscurricular activities with respect to the learning of subject knowledge. As a response to these problems, Hobbs et al. (2019) mention that one of the schools taking part in the Australian initiatives discussed earlier handled the problems of assessment by emphasizing both the students' competence to apply mathematics to real-world problems and their mathematical skills and conceptual knowledge. A recent review (White & Delaney, 2021) of articles that focus on the benefits of crosscurricular STEM and STEAM (STEM and arts) teaching indicates that broadening the focus in assessment can capture a wider array of benefits, including both academic success and motivation. We propose that it could be worthwhile to study the challenges of assessment through the lenses of instrumental and relational views. However, this lies beyond the scope of our chapter.

The evolution of crosscurricular approaches in the Finnish curricula: the case of mathematics

There is a long tradition of public schooling in Finland, and the Finnish educational system has always, to a greater or lesser degree, put emphasis on crosscurricular teaching and learning. Based on a content analysis, this section lays out a brief sketch of crosscurricular approaches, and the position of mathematics within them, in Finnish curricular documents from postwar Finland onward. The section provides an in-depth example, which can be read as an illustration of the negotiation between the interests of contrasting views on mathematics education.

The Finnish Basic Education Act of 1968 stated that all children from the age of 7 should attend a comprehensive basic school, a *grundskola*, for their first nine years of education. Before the 1970s, the Finnish curricula tracked students to "academic" streams or "vocational" streams and there was practically no possibility to move between these streams once students had decided which pathway to follow. The change in postwar Finland during 1945–1970 was from an agricultural nation, where the needs for mathematics in everyday life were foregrounded, to an industrialized society. At this time, mathematics was clearly seen as having an instrumental value in relation to other school subjects and the role of mathematics in any crosscurricular or transcurricular situations was subordinated to the needs of these other subjects. An extreme example is "counting within trade" (*handelsräkning*) which is described as belonging to the "practical subjects" (Kommittébetänkande, 1954: 12, p. 198). However, there are also some indications that a skill in abstract mathematical thinking was seen as a valuable gain on its own, especially when educating students for technical vocations (Kommittébetänkande, 1954: 12, p. 114).

Pedagogical ideas aiming at social gains and more holistic interpersonal development were known in Finland as early as the 1930s, but school education was not greatly influenced by them. This includes the idea of grouping the content of education into thematic, crosscurricular areas – an idea that became a model for the Comprehensive School Curriculum Committee (*Grundskolans läroplanskommitté*). The groundwork for basic schooling for all Finnish pupils, the *grundskola*, was laid by this committee whose visions were published in 1970 in a National Core Curriculum (Kommittébetänkande, 1970: A4), and subject syllabi (Kommittébetänkande, 1970: A5). The overarching curricular vision of the committee was based on the ideas of Bildung, promoting a harmonious development of the individual. The vision further included *vertical integration* within a subject, that is, the internal order of subareas in mathematics within and between grades, as well as *horizontal integration* of the learning content, that is, crosscurricular approaches. The most radical among the suggestions for horizontal integration made by the committee was "to erase boundaries between subjects and gather the subject matter around central problems for students or society" (Kommittébetänkande, 1970: A4, p. 64). The committee's research-based vision for teaching in the new *grundskola* was clearly to implement both crosscurricular and transcurricular approaches: to integrate two or more school subjects, to fuse related subjects, or to merge subjects into new entities or themes. Yet, referring to lack of time and the resistance due to disciplinary interests of stakeholders – "subject experts can hardly free themselves from their subject-centred view" (Kommittébetänkande, 1970: A5, p. 387) – the committee felt compelled to nevertheless build its work on a subject-based curriculum.

Acknowledging the weaknesses of a subject-based curriculum, the committee pointed to the role and responsibility of subject teachers to collaborate and to support students in integrating knowledge and skills holistically and in being initiators in the learning process (Kommittébetänkande, 1970: A5, p. 68). The subject syllabi (Kommittébetänkande, 1970: A5) discusses each subject in terms of cooperation or integration with other school subjects. However, the subject of mathematics is explicitly referred to only in two other subjects: in visual arts and in home economics. Statements in the mathematics syllabus – about individual work, the scarcity of group work, and self-instructional mathematical workbooks – add up to the following conclusion: despite the vision of the Comprehensive School Curriculum Committee for the new *grundskola* concerning Bildung and horizontal integration, mathematical practice was conceived instrumentally, as an individual endeavor, and as a stand-alone subject.

With the National Core Curriculum reform in 1985, under the slogan "a school for all," came the requirement on mathematics teachers to adapt their planning to the same curriculum and syllabus for all students. The visions from 1970 of a more integrated curriculum were however still set as long-term goals for all subjects, including mathematics: "in the planning one should strive to consider the integration of mathematics and other subjects" (Skolstyrelsen, 1985, p. 11). However, the subject-specific content for each grade and the goals for mathematics teaching outlined in the National Core Curriculum were not to be compromised through "collective teaching or interdisciplinary thematic studies" (Skolstyrelsen, 1985, p. 25).

In 1994, it was time for the following curriculum reform of the *grundskola* in Finland (Utbildningsstyrelsen, 1994). One important aim was to reform traditional classroom practices by moving to a more student-centered curriculum, learning how to learn and think, and to increase the possibilities of the schools and teachers to innovate. The 1994 National Core Curriculum was characterized by a remarkable openness, flexibility, and support of creativity and freedom on the school level to use resources, as well as to implement a variation of methods of teaching, a diversity of perspectives on current issues that cross subject boundaries, and a multitude of ways of working cooperatively. There were no division of subject matter between the grades, no set amount of teaching hours per grade, no demand on evidence-based approaches to teaching and learning. It was up to the schools and municipalities to decide to what extent the local curricula would contain instructions on merging subjects into new entities or themes. Hence, in sum, the possibilities in schools to be sensitive to both mathematical subject matter, student interests, and their mutuality were obvious. With requirements to develop mathematical thinking through problem-solving and putting mathematics to use in other subjects came the hope of "making mathematics more fascinating, exciting and surprising" (Utbildningsstyrelsen, 1994, p. 79). Hence, the syllabus of mathematics in the 1994 core curriculum did not deviate from the vision from 1970 of schools working across subject boundaries with the aim of Bildung. However, international evaluators (Norris et al., 1996) were critical of how the 1994 curriculum reform was implemented in practice. In their report, Norris and his colleagues refer to evidence of much traditional whole-class teaching, and the lack of evidence of, "for example, student-centred learning or independent learning," which were two main aims of the reform (Norris et al., 1996, p. 85).

The following National Core Curricula, in 2004 and 2014 (Utbildningsstyrelsen, 2004, 2014), both tried to narrow down differences in local implementations of the national guidelines that was an effect of the 1994 curriculum. The original idea stated by the 1970 Comprehensive School Curriculum Committee of grouping the content of education into thematic, crosscurricular areas reappears in both the 2004 and the 2014 core curricula; in 2004, as a list of seven themes to be integrated into *many* subjects (Utbildningsstyrelsen, 2004). Yet, in the 2004 mathematics syllabus, there are no explicit references to these themes or to other school subjects, and there is hardly any reference to mathematics in the syllabi of other subjects either. In 2014, the idea of crosscurricular and transcurricular approaches reappears as a list of seven interdisciplinary competences to be built up in each subject by applying the content and methods that are typical of that subject (Utbildningsstyrelsen, 2014). The strengthening of Bildung, including the competence to apply mathematics in other school subjects and outside school, is set as the general goal for mathematics teaching. The 2014 mathematics curriculum, hence, latches onto the vision set in the beginning of the 1970s, but mathematics is more clearly than before seen as a vehicle for Bildung purposes. Mathematics teachers are expected to plan for crosscurricular activities while at the same time adhering to the assessment criteria communicated in the syllabus of mathematics. Moreover, this general goal must be juxtaposed with the view of mathematics as a hierarchical subject (one idea leading to another, abstraction building on abstraction) conveyed in statements like "mathematics is a cumulative subject, the teaching of mathematics must therefore proceed systematically" (Utbildningsstyrelsen, 2014, p. 375).

Our conclusion is that the content analysis of the Finnish National Core Curricula and the international cases presented earlier reveal tensions that the mathematics teacher must acknowledge and tackle. In their local implementation of the core curriculum, teachers have to balance the requirements of crosscurricular activities, the integrity of the subject of mathematics, as well as the need to incorporate a broader view on mathematical competence and on assessment. As Drake (2019, p. 88) remarks, "it is very difficult indeed to organize interdisciplinary activities in educational institutions whose very raison d'être is the achievement of pre-determined and specified outcomes." Tytler et al. (2019, p. 77), make an even stronger point: "Historically, an integrated curriculum advocacy has never prevailed against disciplinary interests." However, from the cases studied earlier, we see that there are hints of possible solutions.

Discussion

Considering the tradition of conceiving mathematical practice from an instrumental viewpoint, it comes as no surprise that teachers might relate to the tensions identified earlier by letting the disciplinary interests of mathematics take precedence over the organization of crosscurricular projects. From a Finnish point of view, there are no indications that the school system would be leaving the strong subject-centered curriculum and assessment (Uljens & Rajakaltio, 2017).

Crosscurricular work within schools is not easy for individual teachers regardless of subject affiliation because of the constraints that work against the establishment of a school culture necessary for dealing with such complexity. For example, the organization of school schedules, predetermined curricular structures, high-stake assessments, as well as the daily pressures on teachers' work all impact on the implementation of crosscurricular and transcurricular approaches (Röj-Lindberg et al., 2022). There are also challenges connected to defining the learning goals of the crosscurricular activities – which need not be mathematical in and of themselves – in relation to the learning goals concerning each of the collaborating subjects (Braskén et al., 2019). A successful collaboration between subjects, each bringing viewpoints on the objects of study as well as the methodologies, requires attention to the specific features and complexities of each subject and also to the criteria for evaluating the outcomes of the results of the crosscurricular activities. In the absence of clear assessment criteria, the result is likely to be evaluated in terms of weakly classified generic, or meta-skills criteria such as "learning to learn" (McPhail, 2018). McPhail further points to a danger of allowing curriculum design to be shaped by generic skills and general problems, issues, or projects. He argues that such aspects need to act as pedagogical tools for engagement, but they cannot provide the source for the deeper content itself. The content must instead come from the disciplines if cognitive advancement is to move beyond common sense or the acquisition of generic skills (McPhail, 2018, p. 63). Otherwise, there is a risk that subject-specific knowledge may be used only instrumentally and in isolation, divorced from the wider systems of meaning of which it is a part. This echoes discussions in the Bildung tradition, which has likewise warned against a fragmentation of knowledge and argued for the importance of engaging deeply with specific contents. Concerning mathematics, the question is whether the mathematical concepts applied in the crosscurricular activities are already known or learnt during the activities.

We conclude that if one views mathematics education first and foremost instrumentally, the role of mathematics teaching easily reduces to one of merely providing the quantitative toolbox within a crosscurricular project or theme. If one by contrast views mathematics education relationally – as an activity, as a way of approaching different situations in everyday life, at work, or while doing science and research – a crosscurricular educational context could provide a meaningful, realistic setting in which to engage in doing mathematics and making learners' mathematical knowledge less inert.

References

Barwell, R. (2016). Formal and informal mathematical discourses: Bakhtin and Vygotsky, dialogue and dialectic. *Educational Studies in Mathematics*, *92*(3), 331–345. https://doi.org/10.1007/s10649-015-9641-z

Braskén, M., Hemmi, K., & Kurtén, B. (2019). Implementing a multidisciplinary curriculum in a Finnish lower secondary school – the perspective of science and mathematics. *Scandinavian Journal of Educational Research*, *63*(4), 1–17. https://doi.org/10.1080/00313831.2019.1623311

Charles, R. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Education Leadership*, *8*(1), 9–24.

Cheng, A. K. (2019). *Mathematical modelling for teachers.* Routledge.

Cobb, P., Gresalfi, M., & Hodge, L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education*, *40*(1), 40–68.

Czerniak, C. M., & Johnson, C. C. (2014). Interdisciplinary science teaching. In N. G. Lederman & S. K. Abell (Eds.), *Handbook of research on science education* (Vol. II, pp. 395–411). Erlbaum.

den Braber, N., Krüger, J., Mazereeuw, M., & Kuiper, W. (2019). Mathematics in an interdisciplinary STEM course (NLT) in the Netherlands. In B. Doig, J. Williams, D. Swanson, R. Borromeo Ferri, & D. Drake (Eds.), *Interdisciplinary mathematics education* (pp. 167–183). Springer.

Doig, B., & Jobling, W. (2019). Inter-disciplinary mathematics: Old wine in new bottles? In B. Doig, J. Williams, D. Swanson, R. Borromeo Ferri, & D. Drake (Eds.), *Interdisciplinary mathematics education* (pp. 245–255). Springer.

Doig, B., Williams, J., Swanson, D., Borromeo Ferri, R., & Drake, D. (Eds.). (2019). *Interdisciplinary mathematics education*. Springer.

Drake, P. (2019). Introduction: A glass half full? In B. Doig, J. Williams, D. Swanson, R. Borromeo Ferri, & D. Drake (Eds.), *Interdisciplinary mathematics education* (pp. 85–90). Springer.

Hobbs, L., Doig, B., & Plant, B. (2019). The successful students STEM project: A medium scale case study. In B. Doig, J. Williams, D. Swanson, R. Borromeo Ferri, & D. Drake (Eds.), *Interdisciplinary mathematics education* (pp. 209–227). Springer.

Honey, M., Pearson, G., & Schweingruber, H. (Eds.). (2014). *STEM integration in K – 12 education: Status, prospects, and an agenda for research.* National Academy of Sciences. https://doi.org/10.17226/18612

Kommittébetänkande. (1954: 12). *Kommittébetänkande angående fortsättningsundervisning i landsbygdens folkskolor.* Helsingfors.

Kommittébetänkande. (1970: A4). *Grundskolans läroplanskommittés betänkande I. Grunderna för läroplanen*. Undervisningsministeriet.
Kommittébetänkande. (1970: A5). *Grundskolans läroplanskommittés betänkande II. Läroplan för läroämnena*. Undervisningsministeriet.
Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Ablex.
Liljedahl, P. (2021). *Building thinking classrooms in mathematics*. Corwin SAGE.
McPhail, G. (2018). Curriculum integration in the senior secondary school: A case study in a national assessment context. *Journal of Curriculum Studies*, *50*(1), 56–76. https://doi.org/10.1080/00220272.2017.1386234
Meier, S. L., Cobbs, G., & Nicol, M. (1998). Potential benefits and barriers to integration. *School Science and Mathematics*, *98*(8), 438–445. https://doi.org/10.1111/j.1949-8594.1998.tb17436.x
Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics*, *102*, 9–28. https://doi.org/10.1007/s10649-019-09903-9
Niss, M., & Jensen, T. H. (Eds.). (2002). *Kompetencer og matematiklæring. Ideer og inspiration til udvikling af matematikundervisning i Danmark*. Danish Ministry of Education. http://static.uvm.dk/publikationer/2002/kom/hel.pdf
Norris, N., Aspland, R., MacDonald, B., Schostak, J., & Zamorski, B. (1996). *An independent evaluation of comprehensive school reform in Finland*. National Board of Education.
Röj-Lindberg, A.-S. (2017). *Skolmatematisk praktik i förändring – en fallstudie* [Doctoral dissertation, Åbo Akademi University]. https://urn.fi/URN:ISBN:978-952-12-3618-1
Röj-Lindberg, A.-S., Hartvik, J., & Ingves, M. (2022). En fallstudie om finlandssvenska lärares insikter från ett utvecklingsprojekt. *Nordic Studies in Education/Nordisk Pedagogik*, *42*(2), 176–193. https://doi.org/10.23865/nse.v42.3181
Rowley, C., & Cooper, H. (Eds.). (2009). *crosscurricular approaches to teaching and learning*. SAGE. https://doi.org/10.4135/9781446269282
Schliemann, A. D., Carraher, D. W., & Brizuela, B. M. (2006). *Bringing out the algebraic character of arithmetic. From children's ideas to classroom practice*. Routledge.
Schoenfeld, A. (2017). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). *Journal of Education*, *196*(2), 1–38. https://doi.org/10.1177/002205741619600202
Skemp, R. R. (1978). Relational understanding and instrumental understanding. *The Arithmetic Teacher*, *26*(3), 9–15.
Skolstyrelsen (1985). *Grunderna för grundskolans läroplan*. Skolstyrelsen.
Skovsmose, O., & Valero, P. (2001). Breaking political neutrality: The critical engagement of mathematics education with democracy. In B. Atweh, H. Forgasz, & B. Nebres (Eds.), *Sociocultural research in mathematics education. An international perspective* (pp. 37–55). Lawrence Erlbaum.
Toh, T. L., & Yeo, J. B. W. (Eds.). (2019). *Big ideas in mathematics*. World Scientific Publishing. https://doi.org/10.1142/11415
Tytler, R., Williams, G., Hobbs, L., & Anderson, J. (2019). Challenges and opportunities for a STEM interdisciplinary agenda. In B. Doig, J. Williams, D. Swanson, R. Borromeo Ferri, & D. Drake (Eds.), *Interdisciplinary mathematics education* (pp. 51–80). Springer.
Uljens, M., & Rajakaltio, H. (2017). National curriculum development as educational leadership. In M. Uljens & R. Ylimäki (Eds.), *Bridging educational*

leadership, curriculum theory and didaktik (pp. 3–151). Springer. https://doi.org/10.1007/978-3-319-58650-2_13

Utbildningsstyrelsen. (1994). *Grunderna för läroplanen för grundskolan*. Utbildningsstyrelsen.

Utbildningsstyrelsen. (2004). *Grunderna för läroplanen för den grundläggande utbildningen*. Utbildningsstyrelsen.

Utbildningsstyrelsen. (2014). *Grunderna för läroplanen för den grundläggande utbildningen*. Utbildningsstyrelsen.

Ward-Penny, R. (2011). *Cross-curricular teaching and learning in the secondary school. Mathematics.* Routledge.

White, D., & Delaney, S. (2021). Full STEAM ahead, but who has the map for integration? – A PRISMA systematic review on the incorporation of interdisciplinary learning into schools. *LUMAT Special Issue*, *9*(2), 9–32. https://doi.org/10.31129/LUMAT.9.2.1387

Williams, J., & Roth, W.-M. (2019). Theoretical perspectives on interdisciplinary mathematics education. In B. Doig, J. Williams, D. Swanson, R. Borromeo Ferri, & D. Drake (Eds.), *Interdisciplinary mathematics education* (pp. 13–34). Springer.