Evaluation of experiment designs for MIMO system identification by model predictive control

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Evaluation of Experiment Designs for MIMO System Identification by Model Predictive Control*

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Abstract— **Ten different experiment designs for control-oriented MIMO identification are evaluated by the performance of model predictive control (MPC) using the identified models for control of the true system, which is a moderately ill-conditioned 3×3 system. The evaluated designs are some standard designs as well as advanced designs aimed at ensuring integral controllability requirements. Different types of input perturbations are also considered. Control performance in terms of output error and control activity measures is determined for various choices of sampling time and input constraints. The evaluation clearly shows that for this application, experiment designs where the directionality of the system is taken into account produce better models for control design than more standard methods.**

I. Introduction

In system identification, the experiment design is crucial for obtaining data that are a good representation of the system to be identified. In this respect, multiple-input multiple-output (MIMO) systems are much more challenging than single-input single-output (SISO) systems. Very little is said about the identification of MIMO systems in textbooks on system identification. The most advanced textbook advice for experiment design is that the inputs should be perturbed simultaneously in an uncorrelated way [1, 2].

Control-oriented experiment design has been quite extensively studied in the research literature; see, e.g., [3–6]. However, these studies mostly deal with SISO systems. This paper specifically deals with control-oriented experiment design for MIMO systems, especially ill-conditioned ones. For such systems, there are important issues that are trivial (or lacking) for SISO systems [2]. One such issue is integral controllability, i.e., control of the system using a controller with integral action.

For a SISO system, integral controllability requires that the steady-state gain of the model used for controller design has the same sign as the gain of the true system. For MIMO systems, the requirement is more complex (see Section II). If the possible errors in the gain matrix of the estimated model are randomly distributed, there is a significant possibility that the integral controllability requirement is violated, even if the errors are small [7]. However, if the errors are distributed in a favorable way, integral controllability can be achieved even for quite large errors. Obviously, the distribution of the errors depends on the data. It has been recognized that perturbations that explicitly excite the "gain directions" of the system tend to produce suitable data for a control-oriented MIMO identification [7–10].

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In this paper, ten different experiment designs for MIMO identification are evaluated by model predictive control (MPC) using the identified models. Directional designs as well as more standard designs are considered for various types of perturbations (steps, pulses, PRBS inputs, multi-sinusoidal inputs). The evaluation is based on models identified by realistic simulations of a moderately ill-conditioned 3×3 system. The models are used for model predictive control of the true simulated system with added input and output disturbances. Control performance in terms of output error and control activity measures are considered for various choices of sampling time and input constraints. The conclusions regarding the experiments designs for the considered example are quite clear.

Control-oriented identification of 2×2 systems has been studied quite extensively [10], but detailed studies of larger systems are hard to find, an exception being a 4×4 system studied in [11].

II. Control-Oriented MIMO Identification

A. Integral Controllability

A multivariable controller with integral action can stabilize two systems having gain matrices K and $\hat{K}$, respectively, if and only if [12, 7]

$$\mathrm{Re}[\lambda_i(K\hat{K}^{-1})] > 0 \ , \ \forall i \ , \tag{1}$$

where $\lambda_i(\cdot)$ is the i^{th} eigenvalue of $(\cdot)$. If the system to be controlled has the gain matrix K and the model used for controller design has the gain matrix $\hat{K}$, (1) must hold.

An ill-conditioned system is a MIMO system whose gain matrix has a "high" condition number [13]. Such a matrix is nearly singular, which means that small errors in $\hat{K}$ may result in large errors in $\hat{K}^{-1}$. However, if corresponding column vectors of K and $\hat{K}$ are well aligned, the controllability condition (1) may allow quite large errors in the magnitudes of the column vectors.

It has been recognized that proper excitation of the output directions, especially directions associated with small singular values, tends to produce data that yield a gain matrix with the desired properties [7]. Although the design is based on steady-state considerations only, there are many ways of implementing such a procedure [9]. The fact that slow and fast dynamics of a system tend to be well separated between gain directions makes the method very effective for identification of dynamics, too [10].

B. Directional Input Design

The basic design method for excitation of gain directions is summarized below. It is based on [10] and can be applied to any type of perturbation signal normally used in system identification, such as step changes, pulses, pseudo-random binary sequences (PRBS), and multi-sinusoidal signals.

Consider a system with an input u, an output y, and a non-singular steady-state gain matrix K of size $n \times n$. A singular value decomposition (SVD) of K yields

$$\bar{y} = K\bar{u} = W\Sigma V^{\mathrm{T}}\bar{u} \,, \tag{2}$$

where $\bar{u}$ and $\bar{y}$ denote steady-state values. The matrices V and W are orthogonal and Σ is a diagonal matrix of singular values, σ_i, $i = 1,\ldots,n$, $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n > 0$. The orthogonality means that $V^{\mathrm{T}}V = I$ and $W^{\mathrm{T}}W = I$.

A new signal, the design signal, is defined by

$$\xi(t) = \Sigma V^{\mathrm{T}} u(t) \,, \tag{3}$$

where t denotes continuous or discrete time. The steady-state output is then given by

$$\bar{y} = W\bar{\xi} \,. \tag{4}$$

Because W is given by the SVD, $\bar{\xi}_i$ (i.e., the i^{th} component of $\bar{\xi}$) will excite only the output direction associated with the singular value σ_i resulting in an output with the steady-state magnitude $\| \bar{y} \|_2 = | \bar{\xi}_i |$. The design signal is realized (approximatively) by the true input

$$u(t) = \hat{V}\hat{\Sigma}^{-1}\xi(t) = \sum_{i=1}^{n} \hat{v}_i \hat{\sigma}_i^{-1} \xi_i(t) \,, \tag{5}$$

where $\hat{V}$ and $\hat{\Sigma}$ are estimates of V and Σ, respectively, obtained from an estimate $\hat{K}$ of K. The vector $\hat{v}_i$ is the i^{th} column of $\hat{V}$. If $V^{\mathrm{T}}\hat{V} \approx I$, substitution of (4) into (2) yields

$$\bar{y} \approx W\Sigma\hat{\Sigma}^{-1}\bar{\xi} = \sum_{i=1}^{n} w_i \sigma_i \hat{\sigma}_i^{-1} \bar{\xi}_i \,, \tag{6}$$

where w_i is the i^{th} column of W.

There are numerous design options for ξ. It is possible to excite one gain direction (and the associated dynamics) at a time by perturbing one component ξ_i at a time. Equation (6) shows that an accurate estimate of σ_i is not important as the estimate $\hat{\sigma}_i$ only affects the magnitude of the steady-state output vector $\bar{y}$. It is also possible to excite all gain directions simultaneously by perturbing all components of ξ simultaneously. The perturbations ξ_i should then be uncorrelated with each other to make the various gain directions uncorrelated and thus identifiable. Note that this is different from applying uncorrelated inputs u_i as perturbations.

Independently of the above choice, $\xi_i(t)$ can be any type of signal normally used as input in system identification. If desired, the signals for the various gain directions can be designed with dynamics in mind to excite different frequency ranges. In principle, it is even possible to use different types of signals for the gain directions.

III. Case Study

A. A Moderately Ill-Conditioned 3×3 System

The system for this case study has the transfer function

$$G(s) = \begin{bmatrix} \dfrac{6\,\mathrm{e}^{-5s}}{22s+1} & \dfrac{20\,\mathrm{e}^{-5s}}{337s+1} & \dfrac{-1\,\mathrm{e}^{-5s}}{10s+1} \\ \dfrac{8\,\mathrm{e}^{-5s}}{50s+1} & \dfrac{77\,\mathrm{e}^{-3s}}{28s+1} & \dfrac{-5\,\mathrm{e}^{-5s}}{10s+1} \\ \dfrac{9\,\mathrm{e}^{-5s}}{50s+1} & \dfrac{-37\,\mathrm{e}^{-5s}}{166s+1} & \dfrac{-103\,\mathrm{e}^{-4s}}{23s+1} \end{bmatrix} . \tag{7}$$

The system was originally presented by Vasnani [14], but here an input and an output have been rescaled to make the system more ill-conditioned with parameters rounded to the nearest integer. The condition number of the system is 30.

When MIMO systems are considered, the scaling of the variables is important. When norm-based measures or averages over different signals are used, the signals should be scaled to make equal numerical changes equally significant. It is assumed that the signals, especially the outputs, fulfil these requirements.

B. Identification Experiments and Models

Table I and II show the models identified by the procedures described below. To make the identification realistic, white output noise (covariance 0.2) and low-frequency input disturbances (Schroeder multi-sines with amplitudes 0.01, 0.002, 0.002 for inputs 1, 2, and 3, respectively) were added to the system. The amplitudes of individual input signals (u_i or ξ_i, $i = 1,2,3$) were adjusted to render output vectors of approximately equal 2-norm after which the amplitudes were jointly rescaled to maximize outputs in the approximate range $(-20, 20)$.

MATLAB's System Identification Toolbox [15] was used to identify first-order transfer functions with time delays. Because this study concerns input design evaluation (not identification algorithms), the true system parameters were used as initial values.

1) Step inputs

A step change was applied to each input, one at a time and well separated in time. A model with the transfer function G_{Ss} was identified. The transfer function is shown in the upper left corner of Table I.

A directional step experiment was carried out with the same rescaled step changes applied to ξ_i, $i = 1,2,3$, and the input determined by (5). The estimated gain matrix $\hat{K}$ from the previous experiment was used to obtain the parameters of (5). The model G_{Sds} was identified (lower left corner of Table I).

TABLE I. MODELS IDENTIFIED USING SIMPLE PERTURBATIONS.

Exp.	Step (S)	Pulse (P)
Seq. (s)	$\begin{matrix} \frac{6.0\mathrm{e}^{-5s}}{21s+1} & \frac{23}{476s+1} & \frac{-0.93\mathrm{e}^{-18s}}{52s+1} \\ \frac{7.9\mathrm{e}^{-4s}}{47s+1} & \frac{76\mathrm{e}^{-3s}}{28s+1} & \frac{-5.6\mathrm{e}^{-6s}}{11s+1} \\ \frac{8.9\mathrm{e}^{-6s}}{45s+1} & \frac{-42\mathrm{e}^{-1s}}{194s+1} & \frac{-104\mathrm{e}^{-4s}}{23s+1} \end{matrix}$	$\begin{matrix} \frac{6.1\mathrm{e}^{-5s}}{23s+1} & \frac{11\mathrm{e}^{-5s}}{207s+1} & \frac{-0.57\mathrm{e}^{-12s}}{0.2s+1} \\ \frac{8.2\mathrm{e}^{-5s}}{51s+1} & \frac{77\mathrm{e}^{-3s}}{28s+1} & \frac{-5.6\mathrm{e}^{-4s}}{15s+1} \\ \frac{9.5\mathrm{e}^{-4s}}{53s+1} & \frac{-27\mathrm{e}^{-9s}}{118s+1} & \frac{-103\mathrm{e}^{-4s}}{23s+1} \end{matrix}$
Dir. (ds)	$\begin{matrix} \frac{5.8\mathrm{e}^{-5s}}{22s+1} & \frac{19}{319s+1} & \frac{-0.22\mathrm{e}^{-22s}}{3s+1} \\ \frac{7.8\mathrm{e}^{-6s}}{56s+1} & \frac{76\mathrm{e}^{-3s}}{27s+1} & \frac{-3.6\mathrm{e}^{-2s}}{60s+1} \\ \frac{8.9\mathrm{e}^{-6s}}{37s+1} & \frac{-34\mathrm{e}^{-18s}}{119s+1} & \frac{-101\mathrm{e}^{-5s}}{21s+1} \end{matrix}$	$\begin{matrix} \frac{6.1\mathrm{e}^{-5s}}{22s+1} & \frac{16\mathrm{e}^{-5s}}{225s+1} & \frac{-1.2\mathrm{e}^{-6s}}{4s+1} \\ \frac{7.8\mathrm{e}^{-5s}}{47s+1} & \frac{78\mathrm{e}^{-3s}}{28s+1} & \frac{-5.4\mathrm{e}^{-4s}}{10s+1} \\ \frac{9.3\mathrm{e}^{-5s}}{48s+1} & \frac{-51\mathrm{e}^{-4s}}{258s+1} & \frac{-104\mathrm{e}^{-4s}}{23s+1} \end{matrix}$

TABLE II. MODELS IDENTIFIED USING ADVANCED PERTURBATIONS.

Exp.	PRBS (B)	Multi-Sine (F)
Unc. (u)	$\begin{matrix} \frac{5.8\mathrm{e}^{-5s}}{21s+1} & \frac{37\mathrm{e}^{-7s}}{645s+1} & \frac{-1.1\mathrm{e}^{-6s}}{11s+1} \\ \frac{8.0\mathrm{e}^{-5s}}{51s+1} & \frac{78\mathrm{e}^{-3s}}{28s+1} & \frac{-4.4\mathrm{e}^{-6s}}{8s+1} \\ \frac{9.5\mathrm{e}^{-5s}}{54s+1} & \frac{-38\mathrm{e}^{-5s}}{145s+1} & \frac{-101\mathrm{e}^{-4s}}{22s+1} \end{matrix}$	$\begin{matrix} \frac{6.0\mathrm{e}^{-5s}}{22s+1} & \frac{16\mathrm{e}^{-5s}}{250s+1} & \frac{-1.3\mathrm{e}^{-6s}}{11s+1} \\ \frac{7.7\mathrm{e}^{-5s}}{48s+1} & \frac{77\mathrm{e}^{-3s}}{28s+1} & \frac{-5.6\mathrm{e}^{-4s}}{13s+1} \\ \frac{9.2\mathrm{e}^{-5s}}{50s+1} & \frac{-47\mathrm{e}^{-4s}}{238s+1} & \frac{-105\mathrm{e}^{-4s}}{24s+1} \end{matrix}$
Dir. Unc. (du)	$\begin{matrix} \frac{6.1\mathrm{e}^{-5s}}{23s+1} & \frac{24\mathrm{e}^{-8s}}{433s+1} & \frac{-2.3\mathrm{e}^{-5s}}{46s+1} \\ \frac{8.3\mathrm{e}^{-5s}}{51s+1} & \frac{77\mathrm{e}^{-3s}}{28s+1} & \frac{-5.1\mathrm{e}^{-6s}}{9s+1} \\ \frac{9.6\mathrm{e}^{-5s}}{58s+1} & \frac{-30\mathrm{e}^{-5s}}{118s+1} & \frac{-102\mathrm{e}^{-5s}}{23s+1} \end{matrix}$	$\begin{matrix} \frac{6.0\mathrm{e}^{-5s}}{22s+1} & \frac{19\mathrm{e}^{-5s}}{282s+1} & \frac{-0.93\mathrm{e}^{-6s}}{9s+1} \\ \frac{7.6\mathrm{e}^{-5s}}{46s+1} & \frac{76\mathrm{e}^{-3s}}{28s+1} & \frac{-5.8\mathrm{e}^{-4s}}{11s+1} \\ \frac{8.5\mathrm{e}^{-5s}}{47s+1} & \frac{-37\mathrm{e}^{-5s}}{191s+1} & \frac{-101\mathrm{e}^{-4s}}{23s+1} \end{matrix}$
Dir. Seq. (ds)	$\begin{matrix} \frac{6.0\mathrm{e}^{-5s}}{22s+1} & \frac{16\mathrm{e}^{-4s}}{254s+1} & \frac{-1.1\mathrm{e}^{-4s}}{9s+1} \\ \frac{8.1\mathrm{e}^{-5s}}{50s+1} & \frac{79\mathrm{e}^{-3s}}{29s+1} & \frac{-4.9\mathrm{e}^{-5s}}{9s+1} \\ \frac{8.6\mathrm{e}^{-5s}}{52s+1} & \frac{-39\mathrm{e}^{-6s}}{200s+1} & \frac{-99\mathrm{e}^{-4s}}{23s+1} \end{matrix}$	$\begin{matrix} \frac{6.2\mathrm{e}^{-5s}}{23s+1} & \frac{14\mathrm{e}^{-3s}}{172s+1} & \frac{-1.4\mathrm{e}^{-1s}}{12s+1} \\ \frac{7.8\mathrm{e}^{-5s}}{47s+1} & \frac{78\mathrm{e}^{-3s}}{29s+1} & \frac{-5.5\mathrm{e}^{-4s}}{10s+1} \\ \frac{9.4\mathrm{e}^{-5s}}{48s+1} & \frac{-60\mathrm{e}^{-5s}}{297s+1} & \frac{-105\mathrm{e}^{-4s}}{23s+1} \end{matrix}$

2) Pulse inputs

A double rectangular pulse with switching time $T_{\mathrm{sw}} = 100$ was applied to each input, one at a time and well separated. A model with the transfer function G_{Ps} was identified (upper right corner of Table II)

A directional pulse experiment was carried out with the same rescaled pulses applied to ξ_i, $i = 1,2,3$, and the input determined by (5). The model G_{Pds} was identified (lower right corner of Table I).

3) PRBS inputs

A pseudo-random binary sequence (PRBS) is a deterministic signal with a sequence length N. The signal switches between two levels with a minimum switching time T_{sw} such that the time between switches is some integer multiple of T_{sw}. By design, the sample statistics of the signal accurately mimic white noise [2].

In this case, the minimum switching time $T_{\mathrm{sw}} = 10$ and the sequence length $N = 127$ were chosen. Three uncorrelated PRBS signals were obtained by time-shifting the signals appropriately. They were simultaneously applied to all inputs u_i, $i = 1,2,3$. This is a standard experiment design with uncorrelated PRBS signals. The identified model is G_{Bu} (upper left corner of Table II).

For directional PRBS inputs, the same rescaled signals were applied to ξ_i, $i = 1,2,3$, and the input was determined by (5). This means that all gain directions were excited simultaneously in an uncorrelated way. The identified model is G_{Bdu} (middle left part of Table II).

In another design, the gain directions were excited one at a time. To keep the total experiment length the same, the low-gain direction was excited by a PRBS signal of sequence length $N = 63$, whereas the sequence length $N = 31$ was used for the other gain directions. The identified model is G_{Bds} (lower left corner of Table II).

4) Multi-sinusoidal inputs

A multi-sinusoidal signal has the form

$$u_{\mathrm{F}}(t) = a \sum_{k=1}^{n_{\mathrm{s}}} \cos(\omega_k t + \phi_k), \tag{8}$$

where n_{s} is the number of sinusoids, all (in this case) with the same amplitude a. The individual sinusoids have the frequency ω_k and phase shift ϕ_k, $k = 1,\ldots,n_{\mathrm{s}}$. A so-called Schroeder multi-sine uses the phase shifts [16]

$$\phi_k = -k(k-1)\pi / n_{\mathrm{s}}. \tag{9}$$

Usually, equally spaced frequencies are desired. The frequencies are then calculated by

$$\omega_k = \omega_{\max} k / n_{\mathrm{s}}, \; k = 1,\ldots,n_{\mathrm{s}}, \tag{10}$$

where $\omega_{\max}$ is the highest frequency of interest.

In this case, $n_{\mathrm{s}} = 50$ and $\omega_{\max} = 0.25$ rad/time unit were chosen. The multi-sinusoidal signal then essentially covers the same frequency range as the previously designed PRBS signal. Uncorrelated multi-sinusoidals were obtained by time-shifting $u_{\mathrm{F}}(t)$ appropriately for the three inputs.

Three experiments similar to the PRBS experiments were carried out. This gave the models G_{Fu}, G_{Fdu}, and G_{Fds} (right part of Table II).

IV. EVALUATION BY MPC

A. Setup

The model (7) was implemented in MATLAB & SIMULINK [17] together with a model predictive controller designed by the use of MATLAB's Model Predictive Control Toolbox [18]. The command `mpc` using default options was used for design of controllers based on the models in Table I and II. Various sampling intervals and rate constraints on input changes were tested.

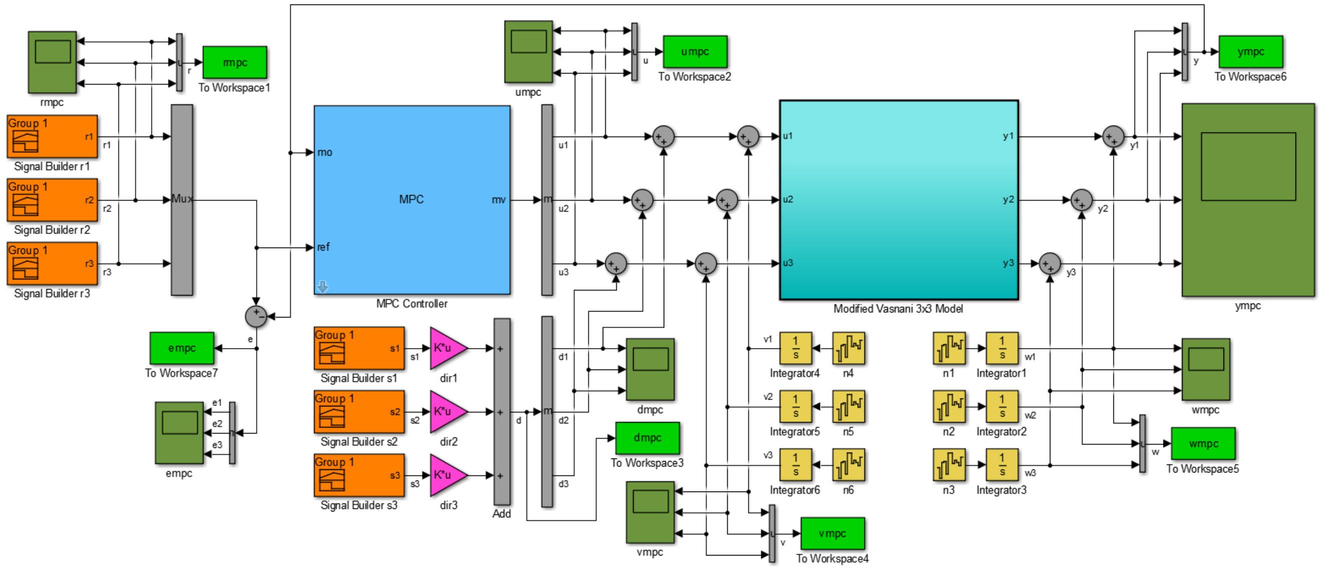

Figure 1. Setup for model predictive control with various input and noise options.

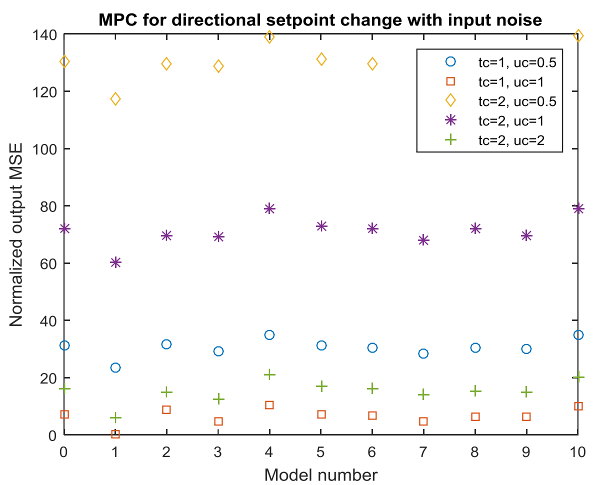

Figure 2. Normalized output MSE for directional setpoint change with brown input noise. Models M, Ss, Sds, Ps, Pds, Bu, Bdu, Bds, Fu, Fdu, Fds (numbers 0,…,10, respectively).

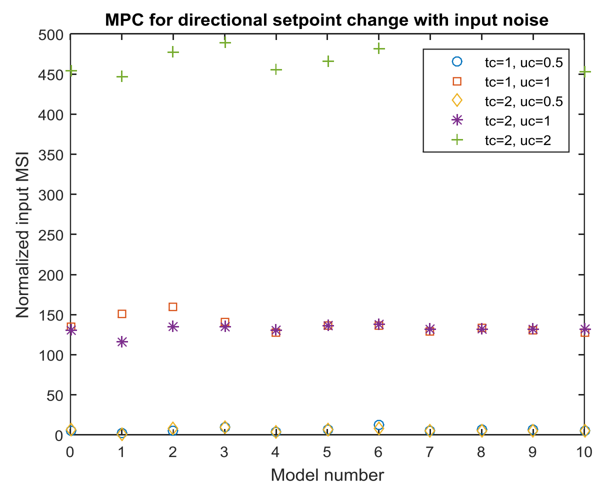

Figure 4. Normalized input MSI for directional setpoint change with brown input noise. Models M, Ss, Sds, Ps, Pds, Bu, Bdu, Bds, Fu, Fdu, Fds (numbers 0,…,10, respectively).

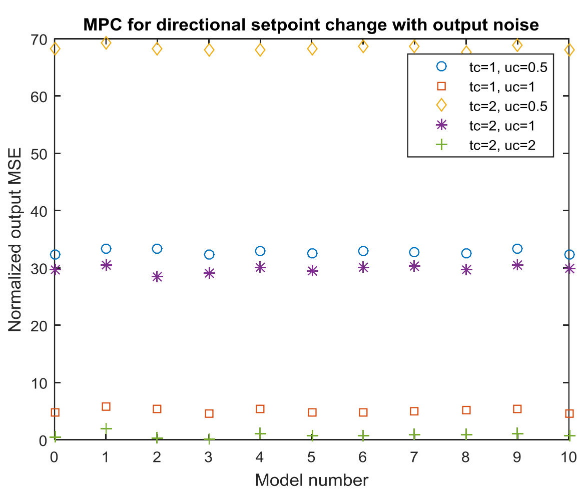

Figure 3. Normalized output MSE for directional setpoint change with brown output noise. Models M, Ss, Sds, Ps, Pds, Bu, Bdu, Bds, Fu, Fdu, Fds (numbers 0,…,10, respectively).

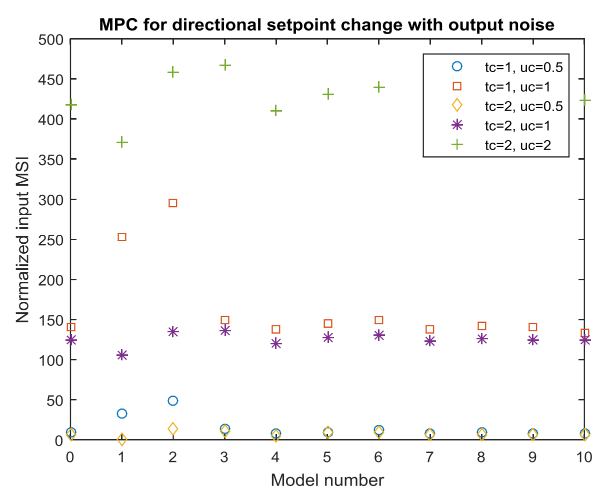

Figure 5. Normalized input MSI for directional setpoint change with brown output noise. Models M, Ss, Sds, Ps, Pds, Bu, Bdu, Bds, Fu, Fdu, Fds (numbers 0,…,10, respectively).

TABLE III. EVALUATION OF IDENTIFIED MODELS FOR INPUT AND OUTPUT NOISE ONLY.

Model	Output MSE	Model	Input MSI
Bds	19.53	Bu	5.98
Ps	21.76	Bds	7.09
Fdu	21.89	Fdu	7.54
Bdu	22.34	Bdu	8.67
Fu	23.38	Fu	11.29
Bu	23.61	Pds	19.79
Fds	28.96	Fds	23.80
Pds	29.32	Ps	29.93
Ss	35.19	Sds	244.60
Sds	41.77	Ss	269.15

TABLE IV. EVALUATION OF IDENTIFIED MODELS FOR SIMPLE SETPOINT CHANGES.

Model	Output MSE	Model	Input MSI
Ss	26.39	Bds	1.42
Bds	28.56	Bu	2.32
Fdu	29.13	Fu	2.35
Ps	29.32	Fdu	2.48
Fu	29.78	Pds	3.88
Bdu	30.73	Fds	5.08
Bu	31.24	Bdu	6.38
Sds	31.71	Ps	8.63
Pds	32.28	Ss	64.72
Fds	32.42	Sds	87.93

TABLE V. EVALUATION OF IDENTIFIED MODELS FOR DIRECTIONAL SETPOINT CHANGES.

Model	Output MSE	Model	Input MSI
Ss	26.85	Pds	4.65
Bds	29.82	Bds	5.16
Ps	30.38	Fds	5.63
Fdu	31.02	Fdu	6.03
Fu	31.12	Bu	6.40
Bdu	31.33	Fu	6.81
Bu	31.58	Ps	9.84
Sds	32.10	Bdu	11.15
Fds	34.00	Ss	11.77
Pds	34.36	Sds	19.74

TABLE VI. EVALUATION OF IDENTIFIED MODELS FOR STEP INPUT DISTURBANCE.

Model	Output MSE	Model	Input MSI
Ss	23.38	Bds	0.00
Bds	26.54	Fds	1.32
Fdu	29.20	Pds	2.75
Fu	31.36	Fu	3.08
Fdu	34.79	Fdu	4.27
Ps	36.74	Bu	7.15
Bu	37.83	Sds	7.81
Bdu	38.16	Bdu	11.13
Pds	38.86	Ps	13.68
Sds	40.41	Ss	23.71

TABLE VII. EVALUATION OF IDENTIFIED MODELS FOR DIRECTIONAL INPUT DISTURBANCE.

Model	Output MSE	Model	Input MSI
Ss	21.22	Pds	0.00
Ps	23.97	Fds	0.87
Bdu	25.60	Bds	2.05
Bds	26.15	Fdu	2.51
Fdu	27.11	Sds	5.17
Sds	29.38	Fu	9.84
Bu	30.93	Ss	11.01
Fu	34.62	Bu	13.16
Pds	41.64	Bdu	13.75
Fds	45.74	Ps	20.73

TABLE VIII. OVERALL EVALUATION OF IDENTIFIED MODELS.

Model	Output MSE	Model	Input MSI
Bds	26.12	Bds	3.14
Ss	26.61	Fdu	4.57
Fdu	27.67	Pds	6.21
Ps	28.44	Fu	6.67
Bdu	29.63	Bu	7.00
Fu	30.05	Fds	7.34
Bu	31.04	Bdu	10.22
Sds	35.07	Ps	16.56
Fds	35.18	Sds	73.05
Pds	35.29	Ss	76.07

The control setup is illustrated by Fig. 1 with various options included. The control performance is studied for simple step changes and directional step changes of the setpoint (r_1, r_2, r_3) as well as disturbances acting on the input (d_1, d_2, d_3). The effect of integrated white noise (i.e., brown noise) on the input (v_1, v_2, v_3) and the output (w_1, w_2, w_3) is also considered. The main evaluation criteria are the mean square deviation from the setpoint of the outputs (MSE) and the mean square control increments of the inputs (MSI).

B. Choice of Sampling Interval and Input Constraints

To obtain control with realistic input changes — and to enhance stability — it is necessary to constrain the inputs. In this case, constraints on RateMin and RateMax (i.e., constraints on incremental input changes) were used.

Figures 2 to 5 illustrate the importance of the sampling interval (t_c) and the input constraints (u_c). The constraint u_c applies to input u_1; for the other inputs, the constraint is $0.1u_c$. For brown input noise, $t_c = 1$ and $u_c = 1$ yield superior performance in terms of output MSE (Fig. 2), whereas $t_c = 2$ and $u_c = 2$ give better performance for brown output noise (Fig. 3). Quite naturally, these give the worst performance in terms of input MSI (Fig. 4 and 5). The best MSI performance is obtained for $u_c = 0.5$. The best compromise seems to be $t_c = 1$ and $u_c = 0.5$.

Other perturbations (simple setpoint change, simple or directional input disturbance) yield very similar results. Thus, the choice of sampling interval and input constraints seem to be more important than the MPC model. Not even the true system (model number 0, denoted M) as MPC model affects this conclusion.

C. Performance of Models

Because of the large variation in performance due to the sampling interval and input constraints, the difference in performance between models for a particular choice of sampling interval and input constrains is not clearly seen in Figures 2 to 5, where the normalized MSE and MSI are given as the deviation from best performance expressed as a percentage. Therefore, the performance for $t_c = 1$ and $u_c = 0.5$ is summarized in Tables III to VIII.

Tables III to VII show normalized mean values of output MSE and input MSI over all noise disturbances considered (input noise, output noise, and both) for no deterministic disturbance (Table III), setpoint changes (Tables IV and V), and deterministic input disturbances (Tables VI and VII). In each table, the performance values are ordered from best to worst with the corresponding model indicated. Table VIII shows overall mean values compiled from Tables III to VII.

As shown by Table VIII, the directional PRBS design Bds is best in terms of both MSE and MSI. Second best, overall, is the directional multi-sinusoidal design Fdu. Sequential step changes (Ss) produce a very good model as evaluated by MSE but the worst according to MSI.

V. Conclusion

Control-oriented experiment designs for identification of MIMO systems, especially ill-conditioned ones, was evaluated by the performance obtained by MPC. A simulated, moderately ill-conditioned, 3×3 system was used for the evaluation. The main evaluation criteria were the mean square error of the outputs (MSE) and the mean square control increments of the inputs (MSI). Various choices of sampling time and input constraints were also considered.

In this study, as well as in some previous ones for systems of other sizes [9, 11], experiment designs that explicitly excited the various gain directions of the system outperformed more standard designs. A directional PRBS design, where each gain direction was excited sequentially, outperformed all other designs in terms of both MSE and MSI. A directional multi-sinusoidal design, where all gain directions were excited simultaneously in an uncorrelated way, was second best.

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