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Input Design to Maximize Information for Identification of MIMO Systems

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***Abstract*— A new input design method for identification of multiple-input multiple-output (MIMO) systems is introduced. The method is completely data based as opposed to previous model-based methods. The data are obtained from one or more experiments with the system to be identified. These experiments do not require any MIMO input design. The design objective is to generate maximally informative data when a new experiment based on MIMO input design is done. The data are considered to be maximally informative when the outputs have maximal variance, subject to some constraints, and no correlation. Since the input design, with given type of perturbation signal, is not unique, it is possible to optimize some additional property, such as minimization of input or output peak value. The various design options are illustrated by application to a system with three inputs and three outputs.**

I. Introduction

The experiment design and its realization are arguably the most important tasks in system identification. If the system is not properly excited by the inputs, the obtained data may not be adequate for model building. To maximize the information content of the outputs, their variances should be at some maximum level with sample correlations as small as possible (ideally zero). This is similar to principal component analysis (PCA), where the score vectors that extract maximum information from a data matrix have this property.

Some model-based experiment design methods have recently been proposed [1–3]. Although the main design criteria are not related to the output distribution, the methods produce nearly uncorrelated outputs [4]. A design method that directly addresses the output distribution was proposed by Häggblom [5]. The optimizations are much simpler than in the previous methods. However, for ill-conditioned systems there might be convergence problems due to the update mechanism used in the iterative solution procedure [6].

In this paper, a data-based design procedure to obtain desired sample properties of the outputs is proposed. Data are obtained from one or more preliminary experiments with the system to be identified. Conditions to obtain uncorrelated outputs with desired variances are given. The proposed method makes it possible to optimize some additional property besides output correlation. In this paper, minimization of input and output peak values subject to the desired output variances are considered.

The various design options are illustrated by an example of an ill-conditioned systems with three inputs and three outputs.

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II. Problem Description

A. MIMO System

Design of experiments for identification of linear multiple-input multiple-output (MIMO) systems is considered. The system has n inputs and n outputs with numerical values $u(k)$ and $y(k)$, respectively, at sampling instant k. Assuming $u(k)$ to be constant between sampling points, the dynamics can be described by

$$y(k) = G(\mathrm{q})u(k)\,, \tag{1}$$

where $G(\mathrm{q})$ is a matrix of pulse transfer operators defined through the shift operator q. This relationship is assumed initially unknown. It is not implied that a model of this form has to be identified.

B. Perturbation Signal

The input design is facilitated by the use of an n-dimensional perturbation signal, $\xi(k)$. Usually, this signal is a random binary sequence (RBS), a pseudo-random binary sequence (PRBS), or a multi-sinusoidal signal (MSS). The individual signal components $\xi_i(k)$, $i = 1,\ldots,n$, should preferably be uncorrelated with one another. To achieve this, the input sequence for ξ_i is taken from one period of a base sequence ξ_0, of length N, by shifting it in a circular way by $(i-1)\lfloor N/n \rfloor$ positions, where $\lfloor N/n \rfloor$ is the largest integer $\le N/n$. More than one period of ξ_0 may be used sequentially in ξ_i [7] to produce a total sequence length n_s.

C. Input Transformation

A simple way to excite the system is to apply perturbations of the inputs simultaneously via $u(k) = \xi(k)$. However, this results in the same amplitudes for the inputs $u_i(k)$, which may not be best. To obtain a good distribution of the outputs $y_i(k)$, $i = 1,\ldots,n$, different amplitudes may be used, or even linear combinations of $\xi_j(k)$, $j = 1,\ldots,n$. This can be done by a linear transformation

$$u(k) = T\xi(k)\,. \tag{2}$$

D. Choice of T Structure

In the design of experiments for identification of MIMO systems, the standard approach, recommended in textbooks on identification [7, 8], is to perturb all inputs simultaneously by mutually uncorrelated signals. This corresponds to a diagonal T and is here referred to as “diagonal inputs”.

To ensure a good identifiability, it is beneficial if the sample correlation between outputs is as small as possible [5, 6]. The output covariance matrix for an $n \times n$ system is defined by $n(n+1)/2$ parameters. This implies that the same number of independent parameters in T is sufficient to obtain uncorrelated outputs. Triangular and symmetric T matrices satisfy this requirement.

A full T matrix results in an underdetermined problem. This makes it possible to optimize some additional condition besides correlation. In this paper, minimization of the peak values $\max_{i,k} |y_i(k)|$ and $\max_{i,k} |u_i(k)|$ is considered.

III. Input Design

The purpose of the input design is to determine the input perturbation $\xi(k)$ and the constant transformation matrix T in (2), which can also be expressed as

$$u_i(k) = \sum_{j=1}^{n} t_{ij} \xi_j(k) \quad , \quad i = 1, \dots, n , \tag{3}$$

where t_{ij} is element (i, j) of T.

A. Design Equations

To streamline the notation, the vector

$$x = \mathrm{vec}(T) \tag{4}$$

is introduced. The vectorization results in $x_\ell = t_{ij}$, where $\ell = i + n(j-1)$. The matrix X is defined

$$X = x^{\mathrm{T}} \otimes I_n , \tag{5}$$

where x^{T} is the transpose of x, $\otimes$ is the Kronecker product, and I_n is the n-dimensional identity matrix.

Assume that the perturbation signal $\xi_j(k)$, $k = 1, \dots, n_{\mathrm{s}}$, is applied to the input $u_i(k)$. This produces an $n_{\mathrm{s}} \times n$ matrix of outputs Y_ℓ, $\ell = i + n(j-1)$, where $y(k)^{\mathrm{T}}$ occupies the k^{th} row. If the system is linear, the n-dimensional input $u(k)$ applied according to (2), will produce an output matrix Y, which can be expressed in terms of Y_ℓ, $\ell = 1, \dots, n^2$, as

$$Y = \sum_{\ell=1}^{n^2} x_\ell Y_\ell = Y_0 X^{\mathrm{T}} , \tag{6}$$

where $Y_0 = [Y_1 \dots Y_{n^2}]$. The output covariance matrix is then

$$P = \sum_{\ell_1=1}^{n^2} \sum_{\ell_2=1}^{n^2} x_{\ell_1} x_{\ell_2} \operatorname{cov}[Y_{\ell_1}, Y_{\ell_2}] = X P_0 X^{\mathrm{T}} , \tag{7}$$

where $P_0 = \operatorname{cov} Y_0$.

B. Data

If an experiment is performed with every combination $u_i(k) = \xi_j(k)$, $i = 1, \dots, n$, $j = 1, \dots, n$, one at a time, every Y_ℓ, $\ell = i + n(j-1)$, is obtained. The overall covariance matrix P_0 can then be calculated and be used in (7) for the input design.

An alternative is to do n experiments with $u_i(k) = \xi_j(k)$, $i = 1, \dots, n$, and j arbitrary (e.g., $j = i$ or $j = 1$). For each experiment, a finite impulse-response (FIR) model can be determined. This makes it possible to simulate all combinations $u_i(k) = \xi_j(k)$ to obtain the required output matrices.

It is also possible to do only one experiment with $u(k) = \xi(k)$, where all inputs are perturbed simultaneously. If the signal components $\xi_j(k)$, $j = 1, \dots, n$, are essentially uncorrelated with one another, it is possible to determine the required FIR models from this single experiment. This kind of experiment is the standard identification experiment recommended in textbooks [7, 8], but here the data is used to design a better experiment.

C. Design Calculations

If the desired output variances are $\operatorname{var} y_i = 1$, $i = 1, \dots, n$, $P = I$ specifies outputs with these variances and no sample correlation. If a structure is chosen with $n(n+1)/2$ independent elements in T such that T is not structurally singular, e.g. a triangular structure, a nonlinear solver can be applied to solve (7) with $P = I$. Because of the quadratic form of (7), there is usually more than one solution satisfying $P = I$.

A convenient way of solving (7) is to linearize it and to solve the linear equation

$$P = X P_0 \tilde{X}^{\mathrm{T}} + \tilde{X} P_0 X^{\mathrm{T}} - \tilde{X} P_0 \tilde{X}^{\mathrm{T}} \tag{8}$$

iteratively. Here $\tilde{X}$ is the solution found in the previous iteration step.

D. Optimizations

It is useful to formulate the calculation as an optimization problem. Then a solution can also be found when T contains fewer than $n(n+1)/2$ independent elements (e.g. a diagonal T) or more elements (e.g. a full T matrix). Useful objective functions to maximize are $\operatorname{trace} P$, $\det P$, and $\underline{\lambda}(P)$, where $\underline{\lambda}(P)$ denotes the minimum eigenvalue of P. The optimizations are done subject to $I - P \succcurlyeq 0$, i.e., $I - P$ is required to be positive (semi)definite.

Maximization of $\operatorname{trace} P$ is useful when $P = I$ cannot be achieved. The optimization will then converge to $P_{ii} = 1$, $i = 1, \dots, n$, if such a solution is possible. This means that the variances are maximized subject to $\operatorname{var} y_i \le 1$. Maximization of $\det P$ and $\underline{\lambda}(P)$ will not necessarily converge to that solution. When $P = I$ is a possible solution, maximization of $\det P$ and $\underline{\lambda}(P)$ will generally converge to such a solution. This follows from the fact that the maximum occurs in both cases when P is diagonal [6, 9].

Because of the existence of multiple solutions satisfying $P = I$, it is possible to optimize some quantity besides covariance. In this paper, minimization of the peak values

$$Y_{\max} = \max_{i,k} |y_i(k)| , \quad U_{\max} = \max_{i,k} |u_i(k)| \tag{9}$$

subject to $P = I$ is considered. Here, Y is given by (6), and

U can be calculated in a similar way. However, more convenient is to use the relationship

$$U = \varXi T^{\mathrm{T}}, \quad \varXi = [\xi(1)\ \xi(2)\ \ldots\ \xi(n_s)]^{\mathrm{T}}. \tag{10}$$

In this work, optimizations are done with the MATLAB software [10] and the YALMIP toolbox [11]. Nonlinear optimizations based on (7) can be done with the MATLAB function fmincon. In YALMIP, the branch-and-bound-solver bmibnb can be used. In addition to fmincon, this solver also uses Gurobi [12].

Maximizing $\operatorname{trace} P$ subject to $P_{ii} \le 1$, $i = 1, \ldots, n$, can be formulated as

$$\max_{T \in \mathcal{T}} \operatorname{trace} P \ \text{ s.t. } \operatorname{diag} P \le \operatorname{diag} I \text{, and (7) or (8),} \tag{11}$$

where $\mathcal{T}$ denotes the allowed structure(s) of T. Using YALMIP, maximizing the minimum eigenvalue $\underline{\lambda}(P)$ can be formulated as

$$\max_{T \in \mathcal{T}} \lambda \ \text{ s.t. } \ \lambda I \preccurlyeq P \preccurlyeq I \ \text{ and (7) or (8).} \tag{12}$$

Minimizing the output peak value $Y_{\max}$ is formulated as

$$\min_{T \in \mathcal{T}} r \ \text{ s.t. } \ -r \le Y \le r \text{, (6), } P = I \text{, and (7) or (8),} \tag{13}$$

and minimizing the input peak value $U_{\max}$ is formulated as

$$\min_{T \in \mathcal{T}} r \ \text{ s.t. } \ -r \le U \le r \text{, (10), } P = I \text{, and (7) or (8).} \tag{14}$$

If (7) is used, a nonlinear solver is needed; if (8) is used, a linear solver is used iteratively.

IV. CASE STUDY

This case study is based on a distillation column model with three inputs and three outputs originally presented by Vasnani [13]. Häggblom [14, 15, 6] rescaled an input and an output, and rounded off the parameters to the nearest integer, to make the system more ill-conditioned, and thus more interesting. The model is

$$G(s) = \begin{bmatrix} \dfrac{6e^{-5s}}{22s+1} & \dfrac{20e^{-5s}}{337s+1} & \dfrac{-1e^{-5s}}{10s+1} \\ \dfrac{8e^{-5s}}{50s+1} & \dfrac{77e^{-3s}}{28s+1} & \dfrac{-5e^{-5s}}{10s+1} \\ \dfrac{9e^{-5s}}{50s+1} & \dfrac{-37e^{-5s}}{166s+1} & \dfrac{-103e^{-4s}}{23s+1} \end{bmatrix}. \tag{15}$$

The condition number of the gain matrix is 30 and the time constants vary by a factor of almost 34.

In this case study, the sampling time 1 is used. The input base sequence ξ_0 is a PRBS with amplitude 1, minimum switching time 5, and sequence length $N = 2^8 - 1 = 255$. One period of the PRBS is used as input. The PRBS was generated by the MATLAB function idinput [16].

A. Standard Input Design

A standard input design, where all inputs are perturbed simultaneously with (essentially) uncorrelated signals, is first illustrated. This is accomplished by a diagonal T matrix. Fig. 1 shows the inputs and the obtained outputs with

$$T = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}. \tag{16}$$

The amplitudes of u_i, determined by t_{ii}, $i = 1, 2, 3$, are selected to obtain approximately equal output variances $\operatorname{var} y_i$. As can be seen, the outputs are strongly correlated, which is not good for identifiability. Fig. 2 displays scatter plots of y_1 vs. y_2, y_1 vs. y_3, and y_2 vs. y_3, which show the correlations even more clearly. The sampled output covariance matrix for this experiment is

$$R_T = \begin{bmatrix} 0.9191 & 0.7266 & 0.8530 \\ 0.7266 & 0.8468 & 0.7812 \\ 0.8530 & 0.7812 & 1.0585 \end{bmatrix}. \tag{17}$$

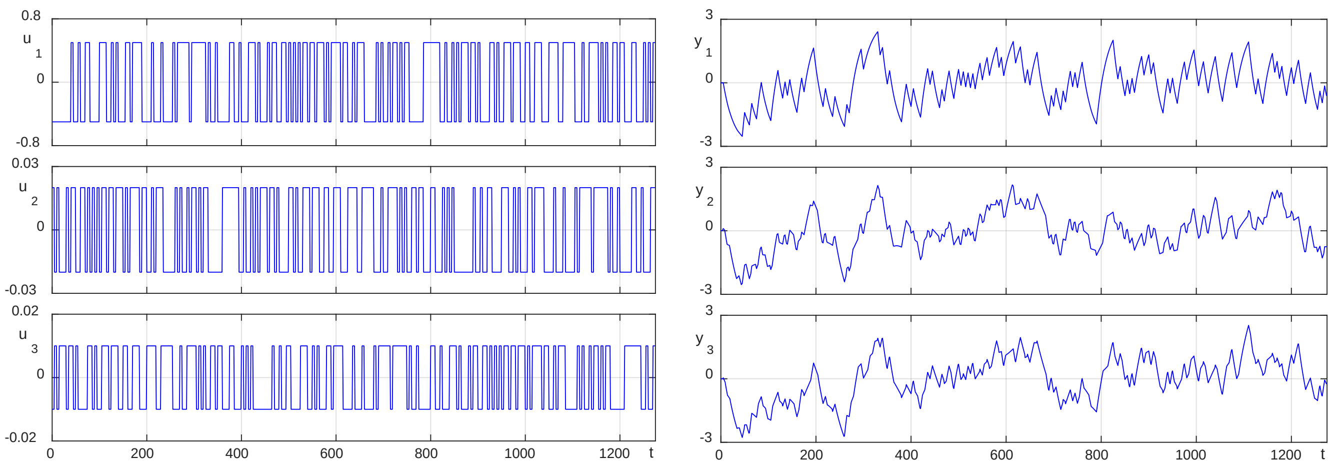

Figure 1. Inputs (left panels) and outputs (rigth panels) of standard input design.

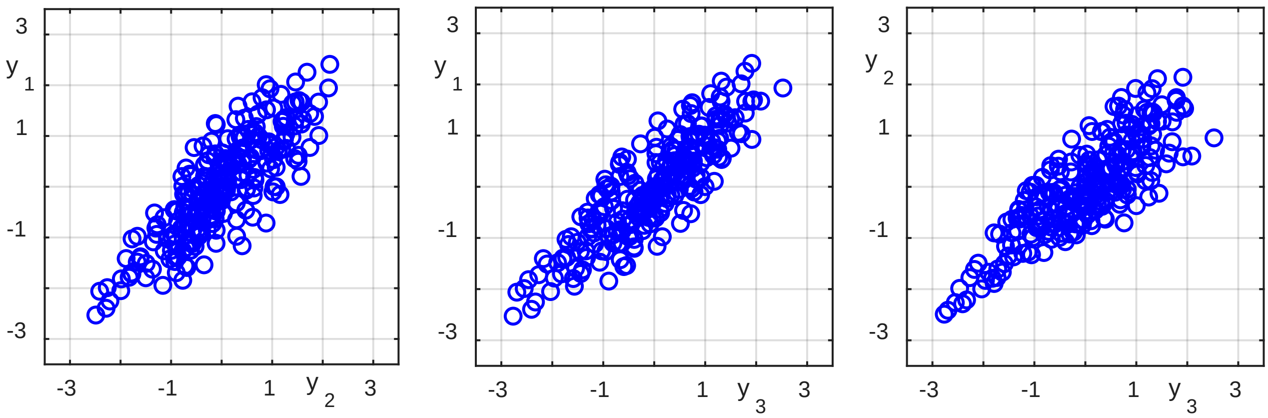

Figure 2. Scatter plots of outputs using standard input design.

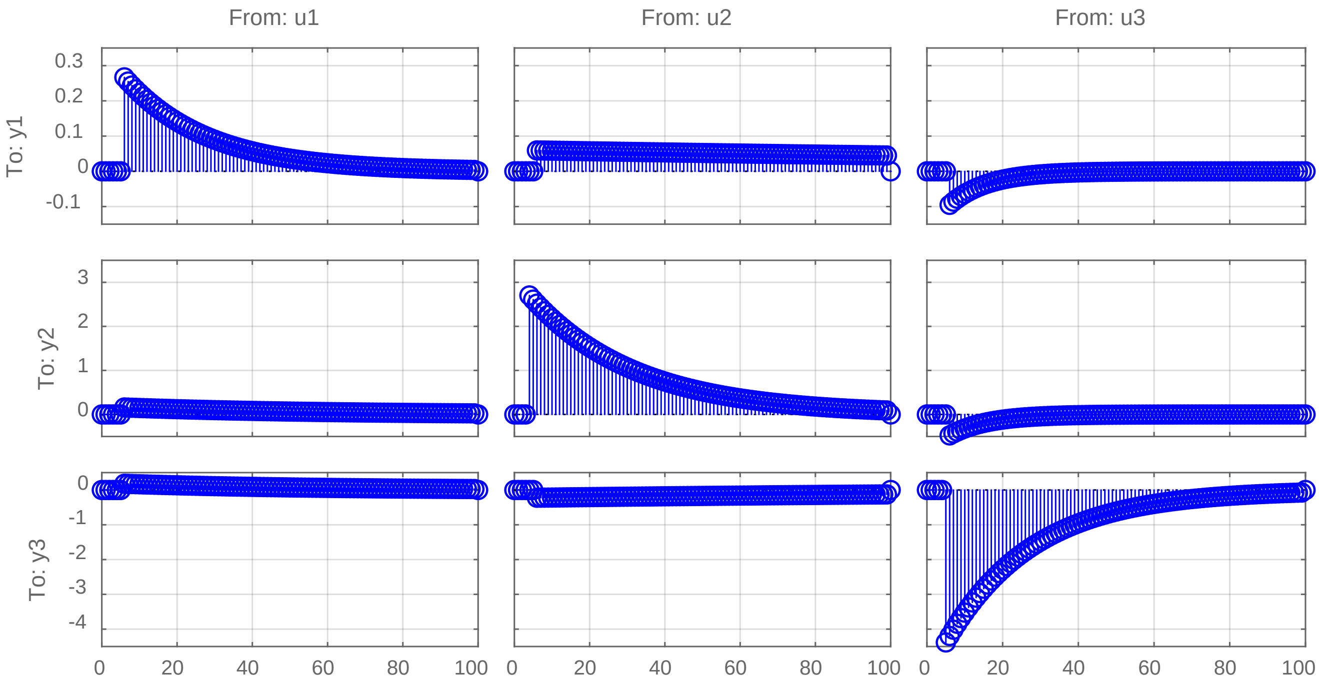

Figure 3. Impulse response coefficients obtained from standard experiment.

B. *Obtaining Data*

The data matrices Y_ℓ, $\ell = i + n(j-1)$, could be determined by simulation of (15) with every combination $u_i(k) = \xi_j(k)$, $i = 1,\ldots,n$, $j = 1,\ldots,n$, one at a time. However, in a practical situation, 9 experiments with the real plant is undesirable.

Fortunately, data from an experiment with the standard input design, which is easy to carry out, can be used to obtain the required data matrices. In this study, the command estimateest, which is available in the Matlab System Identification Toolbox [16], is used to determine FIR models for the 9 input-output relationships.

Fig. 3 illustrates the obtained impulse-response coefficients when the number of coefficients is chosen as $n_h = 100$. The dynamics for all input-output combinations are well captured, except for $y_1 - u_2$ and $y_3 - u_2$, which is due to the slow dynamics of these transfer functions; see (15). However, the gains of these transfer functions are so small that the effect of these errors is negligible.

Simulation of the standard input design (15) with the obtained FIR models results in outputs with the sampled covariance matrix

$$\tilde{R}_T = \begin{bmatrix} 0.9203 & 0.7343 & 0.8557 \\ 0.7348 & 0.8533 & 0.7728 \\ 0.8557 & 0.7728 & 1.0368 \end{bmatrix}, \qquad (18)$$

which is very close to (17). This implies that the matrices Y_ℓ, $\ell = i + n(j-1)$, can be determined with good accuracy by simulation using the obtained FIR models instead of (15).

C. *Triangular T Matrix*

With a triangular T it is possible to achieve $P = I$. There are $(n!)^2$ permutations of triangular-type T matrices, i.e. 36 permutations for a 3×3 system. In this study, only the 6 row permutations of a lower triangular T are considered.

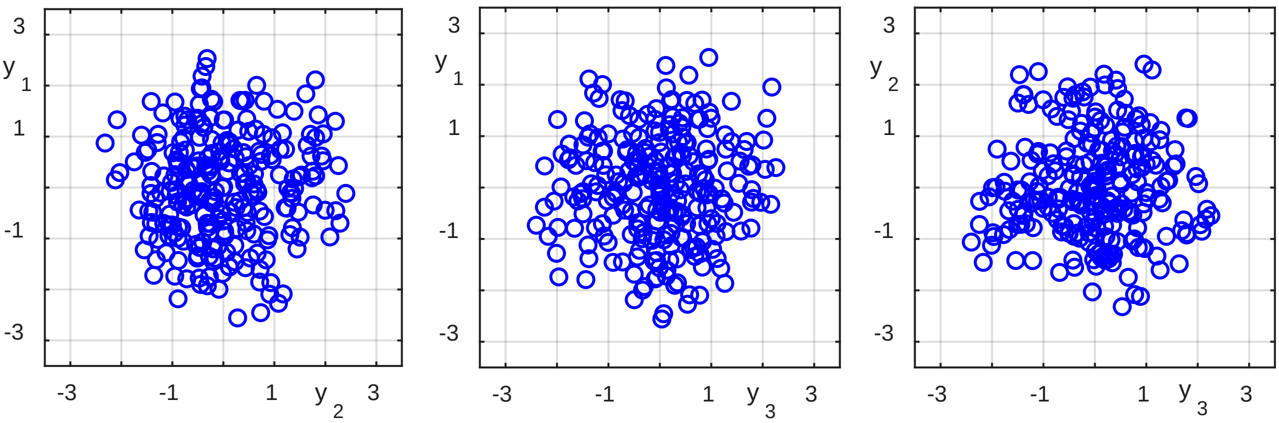

Figure 4. Scatter plots of outputs in triangular design with smallest output peak.

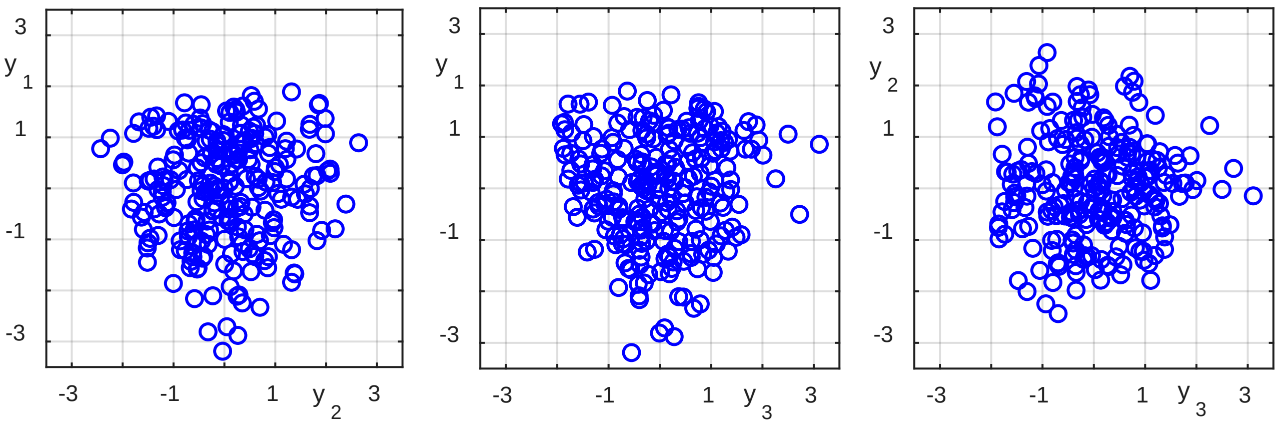

Figure 5. Scatter plots of outputs in triangular design with smallest input peak.

The smallest output peak $Y_{\max} = 2.61$ is obtained with

$$T = \begin{bmatrix} 0.5386 & 0 & 0 \\ -0.0386 & -0.0422 & 0 \\ 0.0305 & -0.0060 & -0.0272 \end{bmatrix}. \qquad (19)$$

The input peak value $U_{\max} = 0.54$. Fig 3. shows output scatter plots of this input design. The largest output peak value $Y_{\max} = 3.59$ is obtained with

$$T = \begin{bmatrix} 0.4225 & 0.1810 & -0.2703 \\ -0.0259 & -0.0510 & 0 \\ 0.0401 & 0 & 0 \end{bmatrix}. \qquad (20)$$

The input peak value $U_{\max} = 0.87$. Fig 4. shows output scatter plots of the input design. In both cases $P = I$ is achieved in the design, i.e., no sample correlation.

D. Full T Matrix

With a full T matrix and the requirement $P = I$, the system is underdetermined. This makes it possible to optimize some additional condition besides output correlation.

Optimization using linearization or fmincon requires a starting guess $T = T_0$. The solution may depend strongly on T_0. Therefore, many starting guesses have to be tried. YALMIP's bmibnb solver does not require an initial guess, but the decision variables have to be explicitly bounded. In this study, the bounds $-0.6 \le t_{ij} \le 0.6$, $\forall i, j$, were used.

Minimizing $Y_{\max}$ by the use of bmibnb yields the design

$$T = \begin{bmatrix} 0.0347 & 0.3872 & 0.3921 \\ 0.0117 & 0.0026 & -0.0546 \\ 0.0257 & 0.0190 & 0.0250 \end{bmatrix}, \qquad (21)$$

which results in $Y_{\max} = 2.46$ and $U_{\max} = 0.81$. Fig 5. shows output scatter plots of the design.

Minimizing $U_{\max}$ by bmibnb yields the design

$$T = \begin{bmatrix} -0.0000 & 0.5360 & -0.0000 \\ -0.0403 & -0.0384 & 0.0090 \\ -0.0116 & 0.0303 & -0.0259 \end{bmatrix}, \qquad (22)$$

which results in $Y_{\max} = 3.15$ and $U_{\max} = 0.54$. Fig 6. shows output scatter plots of the design.

In both cases, the same solution was found by the use of fmincon and extensive testing of starting guesses T_0. Various structures, sign combinations, and element magnitudes with three non-zero elements in T_0 were tested. Optimization using the linearization (8) produced slightly worse solutions with the same set of starting guesses for the minimization of $Y_{\max}$. For minimization of $U_{\max}$, the optimization did not converge using (8) instead of (7).

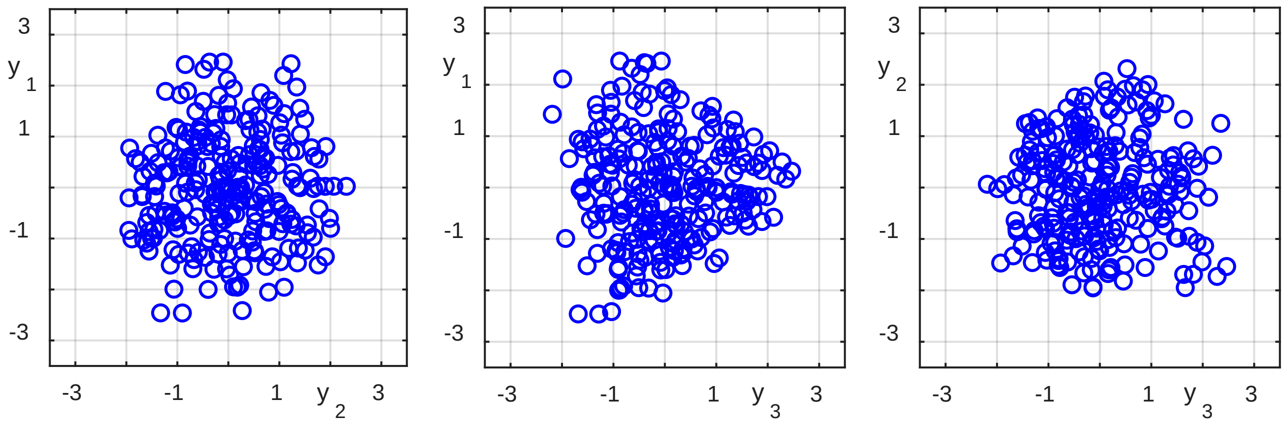

Figure 6. Scatter plots of outputs in full design with minimized output peak.

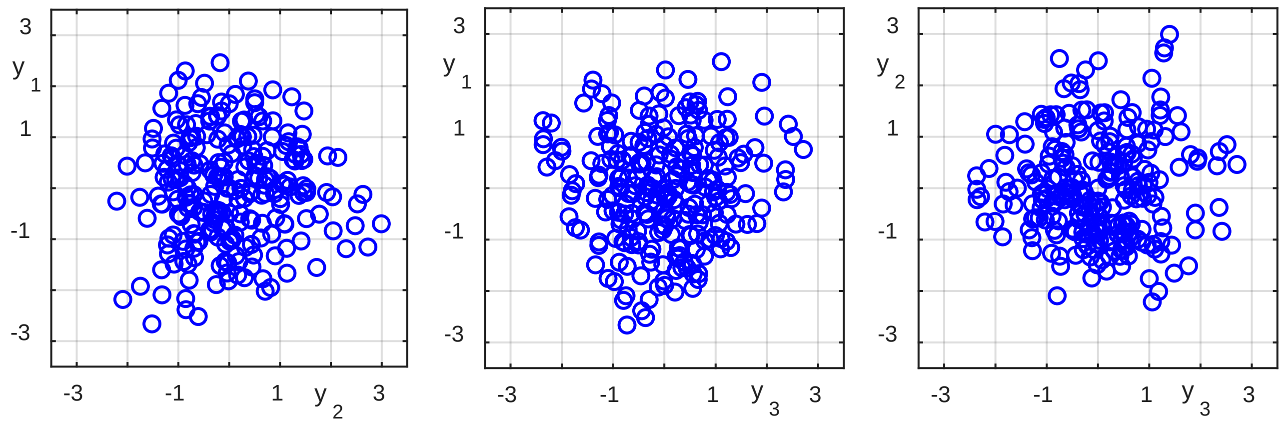

Figure 7. Scatter plots of outputs in full design with minimized input peak.

V. Conclusion

A data-based method for design of experiments for identification of MIMO systems was presented. Previously suggested methods have been model based. The required data can be obtained from a preliminary experiment with the system. The input design yields uncorrelated outputs with desired variances. In analogy with uncorrelated component scores with maximum variance in principal component analysis (PCA), this is considered to maximize the information content of the data. Thus, it is good for identifiability. In addition, input and output peak values can be minimized, which is an advantage in process operation.

References

[1] Rivera, D.E., Lee, H., Mittelmann, H.D. and Braun, M.W. (2007). High-purity distillation: using plant-friendly multisine signals to identify a strongly interactive process. *IEEE Control Syst. Mag.*, 27 (5), 72–89.

[2] Darby, M.L. and Nikolaou, M. (2014). Identification test design for multivariable model-based control: An industrial perspective. *Control Eng. Pract.*, 22, 165–180.

[3] Kumar, A. and Narasimhan, S. (2016). Optimal input signal design for identification of interactive and ill-conditioned systems. *Ind. Eng. Chem. Res.*, 55 (14), 4000–4010.

[4] Häggblom, K.E. (2018). Input designs to obtain uncorrelated outputs in MIMO system identification. *Proc 13th Int. Symp. on Process Systems Engineering – PSE 2018*, 637–642, San Diego, CA, USA.

[5] Häggblom, K.E. (2017). A new optimization-based approach to experiment design for dynamic MIMO identification. *IFAC PapersOnLine*, 50 (1), 7582–7587.

[6] Häggblom, K.E. (2018). Easy ways to design inputs to obtain uncorrelated outputs in MIMO system identification. *IFAC PapersOnLine*, 51 (15), 227–232.

[7] Ljung, L. (1999). *System Identification: Theory for the User*. Prentice Hall, Upper Saddle River, NJ.

[8] Isermann, R. and Münchhof, M. (2011). *Identification of Dynamic Systems*. Springer, Berlin and Heidelberg.

[9] Horn, R.A. and Johnson, C.R. (2013). *Matrix Analysis*, Theorem 7.8.1. Cambridge UP, New York, NY.

[10] The MathWorks, Inc. (2018). *Optimization ToolboxTM User's Guide*. The MathWorks, Inc., Natick, MA.

[11] Löfberg, J. (2004). YALMIP: A toolbox for modeling and optimization in MATLAB. *Proc. IEEE Int. Symp. Computer Aided Control Systems Design (CACSD)*, 284–289, Taipei, Taiwan.

[12] Gurobi Optimization, LLC (2018). *Gurobi Optimizer Reference Manual*. http://www.gurobi.com.

[13] Vasnani, V.U. (1994). Towards relay feedback auto-tuning of multi-loop systems. *PhD Thesis*, National University of Singapore.

[14] Häggblom, K.E. (2015). Evaluation of experiment designs for MIMO system identification by model predictive control. *Proc. IEEE Conf. Control Applications (CCA), Part of 2015 IEEE Multi-Conference on Systems and Control*, 169–174, Sydney, Australia.

[15] Häggblom, K.E. (2016). Evaluation of experiment designs for MIMO identification by cross-validation. *IFAC PapersOnLine*, 49 (7), 308–313.

[16] Ljung, L. (2018). *System Identification Toolbox User's Guide*. The MathWorks, Inc., Natick, MA.