# Evaluation of experiment designs for MIMO identification by cross-validation

Häggblom, Kurt Erik; Skogestad, Sigurd

*Published in:*
IFAC papers online

*DOI:*
10.1016/j.ifacol.2016.07.310

Publicerad: 01/01/2016

Preprint, 11th IFAC Symposium on Dynamics and Control of Process Systems,
including Biosystems
June 6-8, 2016. NTNU, Trondheim, Norway

# Evaluation of Experiment Designs for MIMO Identification by Cross-Validation

**Kurt E. Häggblom**

*Process Control Laboratory, Faculty of Science and Engineering, Åbo Akademi University, FI-20500 Åbo (Turku), Finland (e-mail: khaggblo@abo.fi)*

**Abstract:** Guidelines for design of input perturbations for identification of multivariable systems are given. The focus is on control-oriented identification, but for comparison some general-purpose designs are also considered. The information needed for design of input perturbations such as double rectangular pulses, pseudo-random binary sequences, and multi-sinusoidal signals can be obtained from a simple step test. Such a test also gives information about the directionality properties of the system. This information can be used to design control-oriented experiments as shown in the paper. The design techniques are illustrated by realistic simulations of a moderately ill-conditioned 3×3 system. The identified models, and thus the experiment designs, are evaluated by cross-validations. According to this evaluation, the directional input designs are superior to standard input designs for identification of MIMO systems.

*Keywords:* System identification, Multivariable systems, Ill-conditioned systems, Control-oriented models, Experiment design, Input signals, Validation.

## 1. INTRODUCTION

A successful system identification requires data that are truly representative of the system to be identified. To obtain such data, the experiment design for the identification is of utmost importance. In this respect, multiple-input multiple-output (MIMO) systems are much more challenging than single-input single-output (SISO) systems. Very little is said about the identification of MIMO systems in textbooks on system identification. The most advanced textbook advice for experiment design is that the inputs should be perturbed simultaneously in an uncorrelated way (Ljung, 1999; Isermann and Münchhof, 2011).

Control-oriented experiment design has been studied quite extensively in the research literature; see, e.g., Ljung and Gevers (1986), Bombois et al. (2006), Pronzato (2008), and Hjalmarsson (2009). However, these studies mostly deal with SISO systems. This paper specifically deals with experiment design for MIMO systems, especially ill-conditioned ones.

Figure 1 illustrates a typical problem when uncorrelated inputs are used for identification of a MIMO system. The system has the condition number $\kappa = 10$, which is not particularly ill-conditioned, and the same dynamics in all four transfer functions. As can be seen, the outputs $y_1$ and $y_2$ are strongly correlated, which reduces identifiability. In particular, the so-called low gain direction is poorly excited. Koung and MacGregor (1994) have shown that robust control performance requires a model for control design that accurately models the gain directions of the true system.

In process control, integral control action is usually desired. However, integral control of a MIMO system yields a stable system only if the gain matrix of the model used for control design is "close" to the gain matrix of the true system. The integral controllability requirement may easily be violated if errors in the estimated gain matrix are unfavourable distributed. If the system is ill-conditioned, even very small errors might have this effect. However, if the possible errors are favourably distributed, quite large errors can be tolerated. According to Koung and MacGregor (1993), input perturbations that explicitly excite the various gain directions tend to produce data, where errors are favourably distributed.

In this paper, a general technique for designing "directional" inputs for control-oriented MIMO identification is reviewed. Various options regarding the implementation of the inputs as well as choice of signal types are considered. For comparison, more standard designs are also considered. The methods are illustrated by simulations of a moderately ill-conditioned $3 \times 3$ system. The identified models, and thus the various experiment designs, are evaluated by cross-validations.

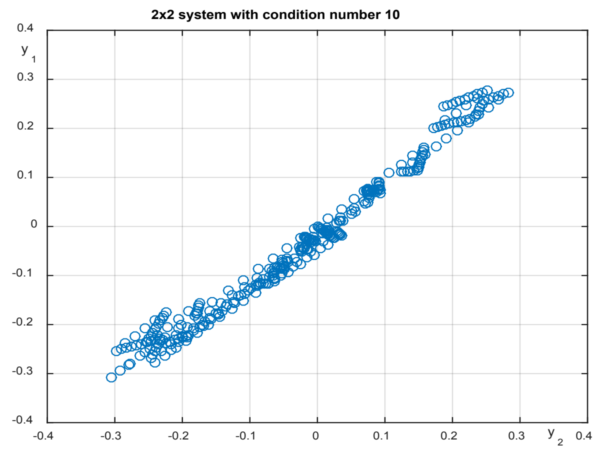


Fig. 1. Outputs of a $2 \times 2$ system with $\kappa = 10$ when inputs are uncorrelated PRBS signals.

## 2. CONTROL-ORIENTED INPUT DESIGN

### 2.1 Integral Controllability and Robust Performance

A multivariable controller with integral action can stabilize two systems having gain matrices $K$ and $\hat{K}$, respectively, if and only if (Garcia and Moarari, 1985; Koung and MacGregor, 1993)

$$\mathrm{Re}[\lambda_i(K\hat{K}^{-1})] > 0 \ , \ \forall i \ , \tag{1}$$

where $\lambda_i(\cdot)$ is the $i$th eigenvalue of $(\cdot)$. If the system to be controlled has the gain matrix $K$ and the model used for controller design has the gain matrix $\hat{K}$, (1) must hold.

An ill-conditioned system is a MIMO system whose gain matrix has a “high” condition number (Skogestad et al., 1988). Such a matrix is nearly singular, which means that small errors in $\hat{K}$ may give large errors in $\hat{K}^{-1}$. However, if the directions of corresponding column vectors in $K$ and $\hat{K}$ are reasonably close to each other, even large errors in the magnitudes of the column vectors can be tolerated without (1) being violated.

Koung and MacGregor (1994) have shown that robust control performance requires $|\,\|\hat{K}K^{-1}\|_2 - 1|$, where $\|\cdot\|_2$ denotes the Euclidean 2-norm, to be small. Essentially, this means that $\|\hat{K}K^{-1}\|_2 \approx 1$ is desired. Under certain assumptions it follows that

$$\|\hat{K}K^{-1}\|_2 = \max_i \left(\frac{\hat{\sigma}_i}{\sigma_i}\right) , \tag{2}$$

where $\sigma_i$ and $\hat{\sigma}_i$ are the $i$th singular values of $K$ and $\hat{K}$, respectively. From (2) it follows, in particular, that the estimate of the smallest singular value must not be grossly in error.

The problems with correlated outputs, integral controllability, and robust performance, can all be tackled by an experiment design such that the various *gain directions are explicitly excited, especially the direction of the lowest gain* (singular value). The basic design methodology (Häggblom, 2014) is outlined in next section.

### 2.2 Directional Input Design

Consider a system with an input $u$, an output $y$, and a non-singular steady-state gain matrix $K$ of size $n \times n$. A singular value decomposition (SVD) of $K$ yields

$$\bar{y} = K\bar{u} = W\Sigma V^{\mathrm{T}}\bar{u} \ , \tag{3}$$

where $\bar{u}$ and $\bar{y}$ denote steady-state values. $V$ and $W$ are orthogonal matrices and $\Sigma$ is a diagonal matrix of singular values, $\sigma_i$, $i = 1,\dots,n$, $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n > 0$. The orthogonality means that $V^{\mathrm{T}}V = I$ and $W^{\mathrm{T}}W = I$.

A new signal, the *design signal* $\xi$, is defined by

$$\xi(t) = \Sigma V^{\mathrm{T}} u(t) , \tag{4}$$

where $t$ denotes continuous or discrete time. The steady-state output is then given by

$$\bar{y} = W\bar{\xi} \ . \tag{5}$$

Because $W$ is given by the SVD, $\bar{\xi}_i$ (i.e., the $i$th component of $\bar{\xi}$) will excite only the output direction associated with the singular value $\sigma_i$ resulting in an output with the steady-state magnitude $\|\bar{y}\|_2 = |\bar{\xi}_i|$. The signal is realized (approximatively) by the true input

$$u(t) = \hat{V}\hat{\Sigma}^{-1}\xi(t) = \sum_{i=1}^{n} \hat{v}_i \hat{\sigma}_i^{-1} \xi_i(t) , \tag{6}$$

where $\hat{V}$ and $\hat{\Sigma}$ are estimates of $V$ and $\Sigma$, respectively, that can be determined from $\hat{K}$. The vector $\hat{v}_i$ is the $i$th column of $\hat{V}$. If $V^{\mathrm{T}}\hat{V} \approx I$, substitution of (6) into (3) yields

$$\bar{y} \approx W\Sigma\hat{\Sigma}^{-1}\bar{\xi} = \sum_{i=1}^{n} w_i \sigma_i \hat{\sigma}_i^{-1} \bar{\xi}_i \ , \tag{7}$$

where $w_i$ is the $i$th column of $W$.

There are many design options for $\xi$. It is possible to excite one gain direction at a time by perturbing one component $\xi_i$ at a time. It is also possible to excite all gain directions simultaneously by perturbing all components of $\xi$ simultaneously. The perturbations $\xi_i$ should then be *uncorrelated* with each other to make the various gain directions uncorrelated and thus identifiable.

Independently of the above choice, $\xi_i$ can be any type of perturbation normally used as input in identification. It can, e.g., be a (series of) step change(s), a double rectangular pulse (DRP), a pseudo-random binary sequence (PRBS), or a multi-sinusoidal signal (MSS). The signals for the various directions can be designed with dynamics in mind to excite different frequency ranges.

Figure 2 shows $y_1$ vs. $y_2$ for the same example as in Fig. 1 when both gain directions are excited simultaneously with uncorrelated PRBS signals $\xi_1$ and $\xi_2$. Now the outputs are much less correlated. It is because of the dynamics that the outputs are not more evenly distributed.

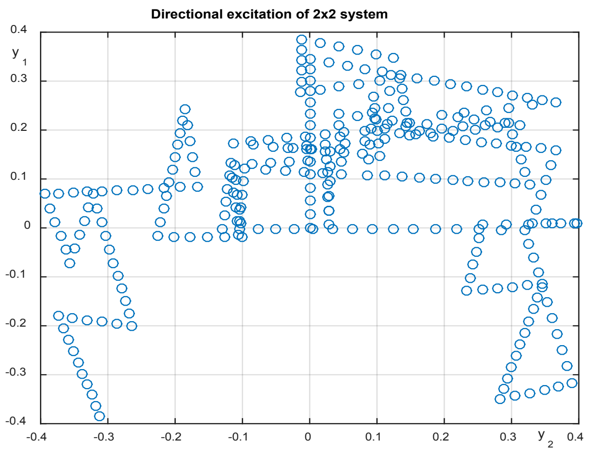


Fig. 2. Directional excitation of a $2 \times 2$ system with $\kappa = 10$.

## 3. DESIGN OF TEST SIGNAL

A proper design of test signal requires some knowledge of the frequency range of interest. This information may be obtained from an initial step test. Such a test will also give an estimate of the gain matrix needed for a directional input design.

In the following, design principles for a DRP, a PRBS, and a MSS are given. They are mainly based on specifications of the desired (closed-loop) bandwidth and the settling time of the system. The signal amplitude $a$ is left as a tuning parameter for desired output magnitudes.

### 3.1 Double Rectangular Pulse

A DRP is composed of a rectangular pulse with height $a$ and duration $T_{\text{sw}}$ directly followed by a similar pulse with height $-a$. The DRP excites almost all frequencies of a system, with the maximum excitation at a higher frequency than the low frequencies mainly excited by a step signal.

There seems to be no readily available design rules for the DRP in the literature. Here we use the simple choice

$$T_{\text{sw}} \approx T_{\text{M}}\,, \tag{8}$$

where $T_{\text{M}}$ denotes the main time constant of interest.

The signal, containing a single DRP starting at some (unspecified) time instant, is denoted $u_{\text{drp}}(t)$. If several inputs are perturbed simultaneously by DRPs, the inputs are

$$u_i(t) = a_i u_{\text{drp}}(t-\theta_i)\,,\ \ i = 1,\ldots,n\,, \tag{9}$$

where $a_i$ is an adjustable amplitude and $\theta_i$ is a suitable time shift in order to apply the DRP to different inputs at different times. For directional inputs, the design signal $\xi_i$ becomes

$$\xi_i(t) = a_i u_{\text{drp}}(t-\theta_i)\,,\ \ i = 1,\ldots,n\ . \tag{10}$$

The true input is calculated by (6).

### 3.2 Pseudo-Random Binary Sequence

A PRBS is a deterministic binary signal with a sequence length $N$, which may be repeated. The signal switches between the levels $a$ and $-a$ with a minimum switching time $T_{\text{sw}}$ such that the time between switches is some integer multiple of $T_{\text{sw}}$. By design, the sample statistics of the signal accurately mimic white noise.

The most commonly used version of a PRBS is a maximum-length PRBS, for which the period length satisfies

$$N = 2^{n_{\text{r}}} - 1\,, \tag{11}$$

where $n_{\text{r}}$ is a positive integer, the so-called register length. With $N$ specified, the switching times can be calculated by a simple formula (Ljung, 1999), but here, the MATLAB System Identification Toolbox (Ljung, 2014) is used.

The design principle is as follows. If the highest angular frequency of interest is $\omega_{\max}$, $T_{\text{sw}}$ should be selected as (Pintelon and Schoukens, 2012)

$$T_{\text{sw}} \approx 2.5\omega_{\max}^{-1}\,. \tag{12}$$

The frequency $\omega_{\max}$ can often be taken as the bandwidth of the system, or of the closed-loop system to be designed.

The period length $NT_{\text{sw}}$ determines the low-frequency excitation. A suitable choice is to select $NT_{\text{sw}}$ as the desired settling time of a step response (Rivera et al., 2007; Isermann and Münchhof, 2011). If the largest time constant of interest is $T_{\text{H}}$, this results in

$$N \approx \beta T_{\text{H}} / T_{\text{sw}}\,, \tag{13}$$

where $\beta$ is chosen according to the desired settling time. For a 95 % settling time, $\beta = 3$; for 99 %, $\beta = 4.6$. Note that $N$ has to be selected to satisfy (11). Compromises might be needed to keep the experiment length $NT_{\text{sw}}$, or some multiple of it, sufficiently short.

The minimum switching time should not be taken as the sampling interval. In fact, it is recommended (Ljung, 1999) that the sampling interval $T_{\text{s}}$ be selected as

$$T_{\text{s}} \approx 0.25T_{\text{sw}}\,. \tag{14}$$

For several inputs, the same PRBS can be used for all inputs provided that it is suitably time shifted for the various inputs to make them statistically uncorrelated. If the PRBS signal is denoted $u_{\text{prbs}}(t)$, the inputs are then

$$u_i(t) = a_i u_{\text{prbs}}(t-\theta_i)\,,\ \ i = 1,\ldots,n\,. \tag{15}$$

Usually the time shifts are selected as $\theta_i = NT_{\text{sw}}(i-1)/n$, $i = 1,\ldots,n$. For directional inputs,

$$\xi_i(t) = a_i u_{\text{prbs}}(t-\theta_i)\,,\ \ i = 1,\ldots,n\,. \tag{16}$$

The true input is calculated by (6).

### 3.3 Multi-Sinusoidal Signal

A MSS has the form

$$u_{\text{mss}}(t) = a\sum_{k=1}^{n_{\text{s}}} \cos(\omega_k t + \phi_k)\,, \tag{17}$$

where $n_{\text{s}}$ is the number of sinusoids, all (in this case) with the same amplitude $a$. The individual sinusoids have the frequency $\omega_k$ and phase shift $\phi_k$, $k = 1,\ldots,n_{\text{s}}$. A so-called Schroeder multi-sine uses the phase shifts (Pintelon and Schoukens, 2012)

$$\phi_k = -k(k-1)\pi / n_{\text{s}}\,. \tag{18}$$

These phase shifts prevent the amplitudes of the sinusoids to add up excessively in the summation.

The remaining user choices are $n_{\text{s}}$ and $\omega_k$, $k = 1,\ldots,n_{\text{s}}$. For $\omega_1 < \omega_2 < \ldots < \omega_{n_{\text{s}}}$, the natural choices for the minimum frequency $\omega_1$ and the maximum frequency $\omega_{n_{\text{s}}}$ are

$$\omega_1 \approx 2\pi/\beta T_{\text{H}}\,,\quad \omega_{n_{\text{s}}} \approx \omega_{\max}\,. \tag{19}$$

Usually, equispaced frequencies are desired. The frequencies are then calculated by

$$\omega_k = 2\pi k / \beta T_{\rm H} \ , \ \ k = 1,\ldots,n_{\rm s} \ , \tag{20}$$

where

$$n_{\rm s} \approx \omega_{\max} \beta T_{\rm H} / 2\pi \ . \tag{21}$$

For several inputs, the same MSS can be used for all inputs provided that it is suitably time shifted for the various inputs to make them statistically uncorrelated. If the MSS is denoted $u_{\rm mss}(t)$, the inputs are

$$u_i(t) = a_i u_{\rm mss}(t-\theta_i) \ , \ \ i = 1,\ldots,n \ . \tag{22}$$

For directional inputs,

$$\xi_i(t) = a_i u_{\rm mss}(t-\theta_i) \ , \ \ i = 1,\ldots,n \ . \tag{23}$$

The true input is calculated by (6).

## 4. SIMULATIONS

### 4.1 Experimental Setup

The system for this case study has the transfer function

$$G(s) = \begin{bmatrix} \dfrac{6\,{\rm e}^{-5s}}{22\,s+1} & \dfrac{20\,{\rm e}^{-5s}}{337\,s+1} & \dfrac{-1\,{\rm e}^{-5s}}{10s+1} \\ \dfrac{8\,{\rm e}^{-5s}}{50\,s+1} & \dfrac{77\,{\rm e}^{-3s}}{28s+1} & \dfrac{-5\,{\rm e}^{-5s}}{10\,s+1} \\ \dfrac{9\,{\rm e}^{-5s}}{50\,s+1} & \dfrac{-37\,{\rm e}^{-5s}}{166\,s+1} & \dfrac{-103\,{\rm e}^{-4s}}{23\,s+1} \end{bmatrix} . \tag{24}$$

The system was originally presented by Vasnani (1994), but here an input and an output have been rescaled to make the system moderately ill-conditioned. The gains and time constants (dimensionless numbers are used) have been rounded to the nearest integer. The condition number of the system is now 30. A SVD of the gain matrix gives

$$\Sigma = \begin{bmatrix} 114 & 0 & 0 \\ 0 & 74.8 & 0 \\ 0 & 0 & 3.78 \end{bmatrix}, \ V = \begin{bmatrix} -0.047 & 0.158 & 0.986 \\ 0.544 & 0.832 & -0.107 \\ 0.838 & -0.532 & 0.125 \end{bmatrix} . \tag{25}$$

For a realistic simulation, output noise, input disturbances, and/or nonlinearities should be added. In this case, low-frequency multi-sines of Schroeder type ($\omega_k = 2\pi k / 1300$, $k = 1,2,3$; $a_1 = 0.01$, $a_2 = a_3 = 0.002$) were added to the inputs. White noise with (approximate) covariance 0.2 was added to the outputs.

The amplitudes of the input signals ($u_i$ or $\xi_i$, $i = 1,2,3$) were adjusted to render output vectors of approximately equal 2-norm. After this, they were jointly re-scaled to maximize outputs in the range $(-20, 20)$. The System Identification Toolbox of MATLAB (Ljung, 2014) was used for all identifications.

### 4.2 Initial Step Experiment

A simple step test was performed to obtain basic information about the system. The inputs were changed one at a time well separated to allow the system to nearly reach a steady state between changes. The input having the fastest dynamics was changed first, slowest dynamics last. The step test is shown in Fig. 3.

The identified model is

$$G(s) = \begin{bmatrix} \dfrac{5.96\,{\rm e}^{-5.5s}}{20.5s+1} & \dfrac{23.4}{476s+1} & \dfrac{-0.93\,{\rm e}^{-18s}}{51.9s+1} \\ \dfrac{7.94\,{\rm e}^{-3.9s}}{47.4s+1} & \dfrac{75.7\,{\rm e}^{-3.2s}}{28.5s+1} & \dfrac{-5.59\,{\rm e}^{-5.8s}}{11.1s+1} \\ \dfrac{8.89\,{\rm e}^{-5.6s}}{44.7s+1} & \dfrac{-42.0\,{\rm e}^{-0.7s}}{194s+1} & \dfrac{-104\,{\rm e}^{-3.5s}}{23.0s+1} \end{bmatrix} , \tag{26}$$

which is reasonable close to the true model. A SVD of the gain matrix gives the estimates

$$\hat{\Sigma} = \begin{bmatrix} 117 & 0 & 0 \\ 0 & 73.6 & 0 \\ 0 & 0 & 3.40 \end{bmatrix}, \ \hat{V} = \begin{bmatrix} -0.044 & 0.161 & 0.986 \\ 0.571 & 0.814 & -0.107 \\ 0.820 & -0.558 & 0.127 \end{bmatrix} . \tag{27}$$

The estimated input directions $\hat{v}_i$, $i = 1,2,3$, are very close to the true ones. This $\hat{\Sigma}$ and $\hat{V}$ are used in the design of all directional inputs.

### 4.3 A Directional Step Experiment

Figure 4 shows a directional step test. The design inputs $\xi_i$, $i = 1,2,3$, are step changes with the amplitudes $a_1 = 20$, $a_2 = 15$, and $a_3 = 10$, occurring at $t = 800$, 400, and 0, respectively. Thus, the low-gain direction is excited first, the high-gain direction last. The true input is calculated by (6).

### 4.4 Double Rectangular Pulse Experiments

For pulse tests, the switching time $T_{\rm sw} = 100$ was used. Figure 5 shows an experiment with non-directional inputs and Fig. 6 an experiment with directional inputs.

### 4.5 PRBS Experiments

For PRBS tests, the minimum switching time $T_{\rm sw} = 10$ and the sequence length $N = 127$ were chosen. This corresponds to a settling time $NT_{\rm sw} = 1270$. The sampling interval given by (14) was rounded to $T_{\rm s} = 2$. This sampling interval was used in all experiments. Figure 7 shows an experiment with uncorrelated PRBS inputs designed in this way.

Two types of directional PRBS experiments were made. Figure 8 shows an experiment where all gain directions are excited simultaneously. In Fig. 9, the gain directions are excited one at a time. The low-gain direction is excited first with a sequence length $N = 63$, after which the other directions are excited with sequence lengths $N = 31$. This means that the lowest excited frequencies are, approximately, two and four times higher than in the experiment with uncorrelated PRBS inputs.

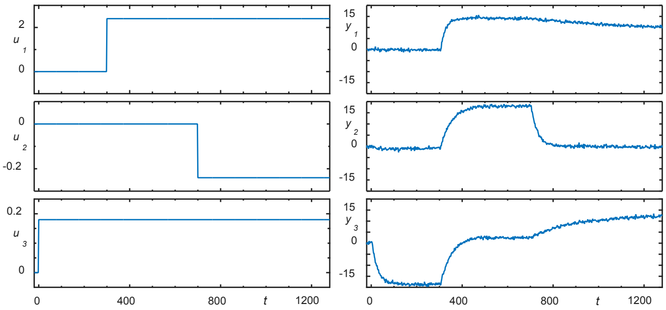


Fig. 3. Initial step experiment (Step).

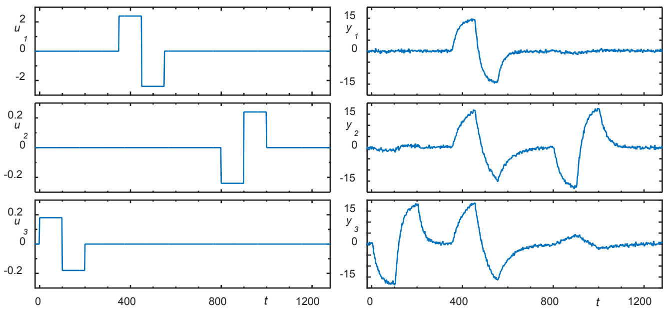


Fig. 5. Non-directional pulse experiment (Pulse).

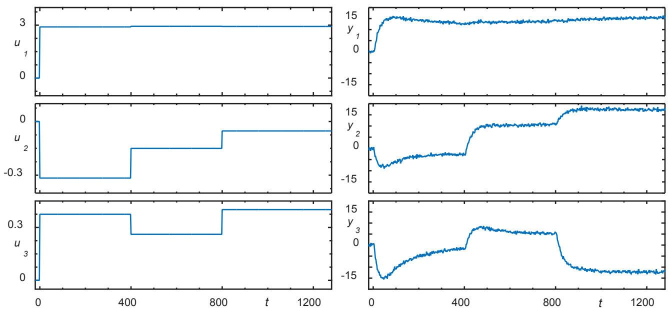


Fig 4. Directional step experiment (StepD).

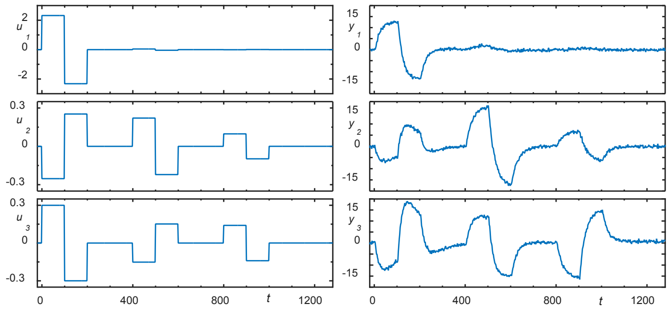


Fig. 6. Directional pulse experiment (PulseD).

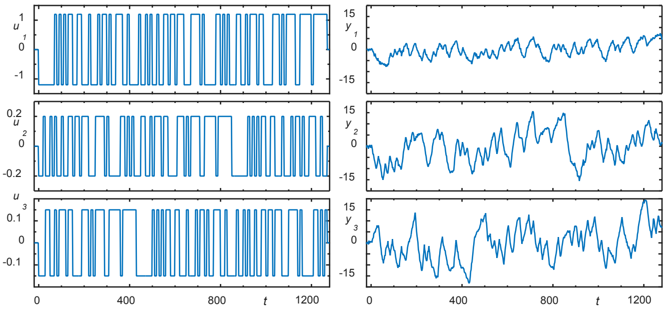


Fig 7. Uncorrelated PRBS experiment (Prbs).

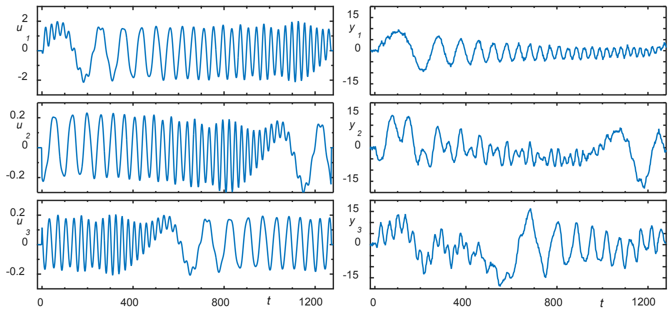


Fig 10. Uncorrelated multi-sine experiment (Mss).

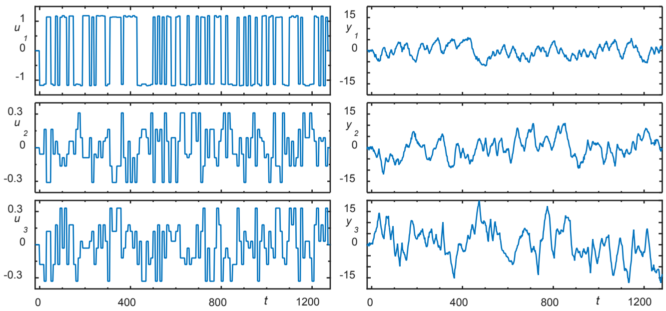


Fig 8. Direction PRBS experiment (PrbsD).

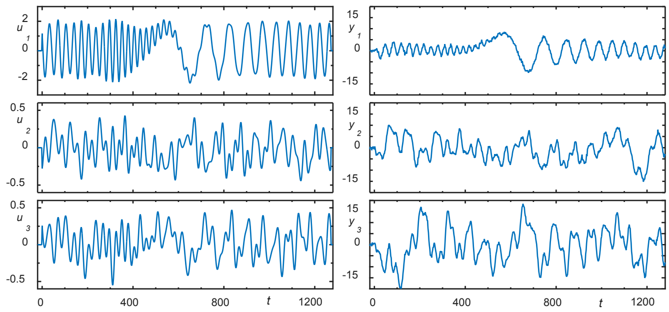


Fig. 11. Directional multi-sine experiment (MssD).

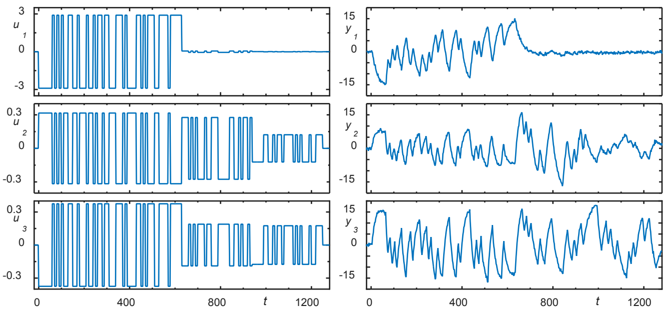


Fig 9. Directional PRBS experiment in sequence (PrbsDs).

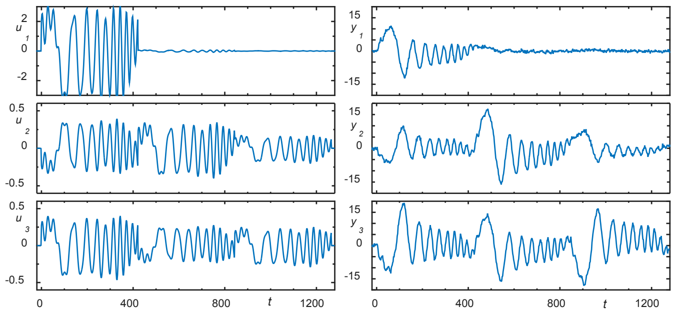


Fig. 12. Directional multi-sine exp. in sequence (MssDs).

### *4.5 Multi-Sinusoidal Experiments*

For the multi-sinusoidal experiments, the period length was chosen as $\beta T_{\mathrm{H}} = NT_{\mathrm{sw}} = 1270$ and the maximum frequency as $\omega_{\max} = 2.5 / T_{\mathrm{sw}} = 0.25$, both in accordance with the PRBS designs. This resulted in $n_{\mathrm{s}} = 50$ frequencies. The time shifts $\theta_1 = 0$, $\theta_2 = 304$, and $\theta_3 = 468$ were found to give uncorrelated inputs. Figure 10 shows an experiment with uncorrelated multi-sinusoidals designed in this way.

Two types of directional multi-sinusoidal experiments were made. Figure 11 shows an experiment where all gain directions are excited simultaneously. In Fig. 12, the gain directions are excited one at a time with period lengths 420 using 17 sinusoids. The lowest excited frequency is three times larger than in the previous experiment.

## 5. CROSS-VALIDATION

Because of space limitations, the identified models are not shown. However, they are reported in Häggblom (2015), where they were used for evaluation of experiment designs by means of model predictive control (MPC). In that study, it turned out that other issues than the choice of model tend to be more important in MPC.

Here, the models, and thus the experiment designs, are evaluated by cross-validation (CV). Each model is used to predict the outputs of all other experiments, given the inputs of those experiments. Table 1 shows the average CV fits in terms of normalized root mean square error (NRMSE) percentages as given by the identification toolbox. The models, and thus the experimental designs, are ordered from best to worst according to the CV fit. The normalization makes the CV fits appear close to each other, but in reality the differences are substantial when plots are compared.

For comparison, the model fits are also given as NRMSE percentages. Note that a good model fit does not guarantee a good model as illustrated by the Prbs and MssDs models.

A similar cross-validation evaluation was made in Häggblom and Böling (2013) on experimental data from a pilot-scale distillation column.

**Table 1. Evaluation of identified models.**

| Model | CV Fit | Mod. Fit |
|---|---|---|
| PrbsDs | 90.36 | 92.25 |
| MssD | 89.83 | 90.53 |
| PrbsD | 89.67 | 89.05 |
| Mss | 89.28 | 91.02 |
| Step | 88.43 | 94.04 |
| PulseD | 88.11 | 92.34 |
| Pulse | 88.05 | 92.71 |
| StepD | 87.95 | 87.79 |
| Prbs | 86.19 | 90.89 |
| MssDs | 86.07 | 90.13 |

## 6. CONCLUSIONS

Experiment designs for MIMO system identification were evaluated by evaluating the identified models using cross-validation techniques. The control-oriented designs based on directional input perturbations using PRBS and multi-sinusoidal signals were found to be superior to other designs. Excitation by uncorrelated PRBS signals, which is the standard suggested design, was especially inefficient.

## REFERENCES

Bombois, X., Scorletti, G., Gevers, M., Van den Hof, P.M.J., and Hildebrand, R. (2006). Least costly identification experiment for control. *Automatica*, 42 (10), 1651–1662.

Garcia, S.E. and Morari, M. (1985). Internal model control. 2. Design procedure for multivariable systems. *Ind. Eng. Chem. Process Des. Dev.*, 24 (2), 472–484.

Häggblom, K.E. (2014). On experiment design for identification of ill-conditioned systems. In *Proc. 19th IFAC World Congress*, 1428–1433, Cape Town, South Africa.

Häggblom, K.E. (2015). Evaluation of experiment designs for MIMO system identification by model predictive control. In *Proc. IEEE Conf. Control Applications* (CCA), 169–174, Sydney, Australia.

Häggblom, K.E. and Böling, J.M. (2013). Experimental evaluation of input designs for multiple-input multiple-output open-loop system identification. In *Proc. IASTED Int. Conf. Control and Applications*, 157–163, Honolulu, HI, USA.

Hjalmarsson, H. (2009). System identification of complex and structured systems. *Eur. J. Control*, 15 (3–4), 275–310.

Isermann, R. and Münchhof, M. (2011). *Identification of Dynamic Systems*. Springer, Berlin and Heidelberg.

Koung, C.-W. and MacGregor, J.F. (1993). Design of identification experiments for robust control. A geometric approach for bivariate processes. *Ind. Eng. Chem. Res.*, 32 (8), 1658–1666.

Koung, C.-W. and MacGregor, J.F. (1994). Identification for robust multivariable control: The design of experiments. *Automatica*, 30 (10), 1541–1554.

Ljung, L. (1999) *System Identification: Theory for the User*. Prentice Hall, Upper Saddle River, NJ.

Ljung, L. (2014). *System Identification Toolbox™ User's Guide*. The MathWorks, Inc., Natick, MA.

Ljung, L. and Gevers, M. (1986). Optimal experiment designs with respect to the intended model application. *Automatica*, 22 (5), 543–554.

Pintelon, R. and Schoukens, J. (2012). *System Identification: A Frequency Domain Approach*. Wiley, Hoboken, NJ.

Pronzato, L. (2008). Optimal experiment design and some related control problems. *Automatica*, 44 (2), 303–325.

Rivera, D.E., Lee, H., Mittelmann, H.D., and Braun, M.W. (2007). High-purity distillation. *IEEE Control Syst.*, 27 (5), 72–89.

Skogestad, S., Morari, M., and Doyle, J.C. (1988). Robust control of ill-conditioned plants: High-purity distillation. *IEEE Trans. Autom. Control*, 33 (12), 1092–1105.

Vasnani, V.U. (1994). Towards relay feedback auto-tuning of multi-loop systems. PhD Thesis, National University of Singapore.