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EFFICIENT IDENTIFICATION ALGORITHM FOR CONTROLLING MULTIVARIABLE TUMOR MODELS: GRADIENT-BASED AND TWO-STAGE METHOD

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Abstract. This paper presents Gradient-based Iterative (GI) and Two-Stage Gradient-based Iterative (2S-GI) identification algorithms for the Controlled Auto-Regressive Moving Average (CARMA) form of a multivariable tumor model. The mathematical proof of the 2S-GI algorithm for multivariable CARMA systems is provided, demonstrating its effectiveness in parameter estimation. The step-by-step introduction of the algorithm facilitates further studies and implementation. A comprehensive comparison between the GI and 2S-GI algorithms is conducted, evaluating their performance in terms of convergence rate and estimation accuracy. The introduced multivariable tumor model serves as a testbed for the algorithms' effectiveness. The results of the comparison, supported by simulated data, demonstrate the superiority of the 2S-GI algorithm in accurately estimating the parameters of the CARMA system. This research provides valuable insights into the application of gradient-based iterative algorithms in controlling multivariable tumor models, paving the way for improved control strategies in cancer treatment.

Keywords: Multivariable identification, Parameter estimation, Tumor model, Controlled auto-regressive moving average (CARMA) model, Cancer treatment.
AMS Subject Classification: 39A05,37-04.
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1 Introduction

During the last few decades, industrial systems demanded a suitable model of the complex and complicated systems and as a matter of fact, system identification and parameter estimation have gained huge respect among engineers (Kerschen et al., 2006; Katayama, 2005; Paraskevopoulos, 2017; Eykhoff, 2014). It should be mentioned that mathematical models have gained attention in the last few years because of the paramount role they play in saving time, effort and cost (Watson et al., 2022; Khan & Atangana, 2022). The iterative parameter estimation approaches are suitable for taking advantage of ample input and output data and have the ability to foster the parameter estimation precision (Xu, 2022; Lu et al., 2022).

In control engineering, there are manifold multi-variable plants with complicated construction and perturbation containing uncertainty, having multi-input multi-output systems, multi-input single-output systems, and single-input multi-output systems (Schwedersky et al., 2022; Fazzi et al., 2022). In this contribution, we only concentrate on systems with Multi-Input and

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Multi-Output (MIMO) and try to extract methods that could estimate the MIMO systems parameters.

CARMA models have gined considerations for many scholars (Raja & Chaudhary, 2015). In the domain of CARMA for MIMO systems, many studies have been done (Rui & Li, 2022; Janjanam et al., 2022). For instance, considering CARMA for MIMO systems, in Lyu et al. (2022) the system was divided into two subsystems which there are linear and nonlinear subsystems, and parameters of the model of noise were detected throughout the procedure. Also, GI and Least Squares Based Iterative (LSI) methods for estimation of the parameter of Controlled Auto-Regressive Auto-Regressive moving average (CARARMA) MIMO systems are presented in Ding et al. (2012) and as a novelty in this manuscript 2S-GI approach is developed for CARMA model of MIMO systems. In Fatimah & Joshi (2021) a new method with a high convergence rate and minimal computational cost is proposed for the system identification of the auto-regressive moving average or ARMA model of MIMO systems. It should be mentioned that the CARMA model is more complicated than the ARMA model.

In Ding et al. (2014), 2S-GI were applied to a Single-Input Single-Output (SISO) CARARMA system while in this paper 2S-GI is used to estimate the parameters of a multi-input multi-output system as a novelty. In summary, there are currently available LSI and GI methods for system identification of the CARMA model in MIMO systems. However, there is a research gap regarding the application of the 2S-GI algorithm to MIMO systems, despite its previous utilization in SISO systems several years ago.

In the procedure of tumor modeling, many mathematical models both with multiple and single parameters are investigated. In Adam (1986), a simple SISO model is brought up which has benefits for experimental and theoretical issues. In Yin et al. (2019), a few models are illustrated which include partial differential, algebraic, and ordinary differential equations. Also, a whole-body nanoparticle pharmacokinetics predictive mathematical model and their delivery of tumor are developed in Dogra et al. (2020).

Beyond that, due to the fact that there has been no research accomplished in the realm of estimating the CARMA system parameters in the field of MIMO models with a 2S-GI method, in this article, we aim to represent one novel method for estimating parameters of a CARMA model format by direct usage of existing GI method and developing its 2S-GI algorithms. First, we introduce the mathematical terms of a generic CARMA model system. After that, the two GI methods, in which one of them is old (GI) and one of them is new (2S-GI), are presented mathematically and both of them are illustrated step-by-step in a simple configuration for the reader, so as to make it possible for further use. Novelties of this contribution are listed below:

- Mathematical proof of 2S-GI algorithms for multivariable CARMA systems,
- Introduction of the step-by-step algorithm of 2S-GI algorithms for multivariable CARMA systems,
- Showing effectiveness and convergence rate of the presented approach for estimating multivariable CARMA system parameters by comparing it to the GI method,
- An important tumor model has been identified by introduced identification approaches.

The rest of the paper is formed as demonstrated: In the following part of the paper, a specified description of the model of the system related to the CARMA model for multivariable systems is provided. Section 3 includes all the necessary mathematics of two GI algorithms. Section 4 represents an advantageous tumor model. In Section 5, all the necessary simulations for showing the efficacy of new algorithms are brought up. In Section 6, some of the results are investigated and some discussions are added. Finally, at section 6, all the conclusions are derived.

2 CARMA configuration

Take the following CARMA system into consideration

$$A(z)y(t) = B(z)u(t) + C(z)\nu(t), \tag{1}$$

where $u(t) \in \mathbb{R}^r$ is the input sequence, $y(t) \in \mathbb{R}^m$ is the output sequence of the system, and $\nu(t) \in \mathbb{R}^m$ is a sequence of white noise with variance σ^2 and zero means. Also, $A(z)$, $B(z)$ and $c(z)$ are polynomials in the unit backward shift operator [i.e. $z^{-1}u(t) = u(t-1)$]. For simplicity in the rest of the paper, we have the following notations: $A =: X$ symbolizes A is manifested as X; The character I (I_n) is an identity matrix of suitable size ($n \times n$); 1_n denotes an n-dimensional column vector which every component of it is 1. The superscript T represents the transpose of the matrix; the matrix X norm has the meaning that $\|X\|^2 = tr(XX^T)$.

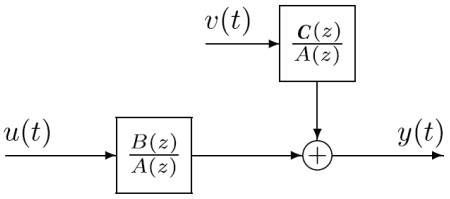

Figure 1: CARMA model system

Now look at the CARMA system shown in Figure 1. We define $A(z)$, $B(z)$ and $C(z)$ as polynomials of known orders (n_a, n_b, n_c) as follows

$$\begin{aligned} A(z) &:= I + A_1 z^{-1} + A_2 z^{-2} + ... + A_{n_a} z^{n_a}, \\ B(z) &:= B_1 z^{-1} + B_2 z^{-2} + ... + B_{n_b} z^{n_b}, \\ C(z) &:= I + C_1 z^{-1} + C_2 z^{-2} + ... + C_{n_c} z^{n_c}. \end{aligned}$$

In a general way, it is assumed that $y(t) = 0$, $u(t) = 0$ and $\nu(t) = 0$ for $t \leq 0$. Note that $A_i = \mathbb{R}^{m\times m}$, $B_i = \mathbb{R}^{m\times r}$ and $C_i = \mathbb{R}^{m\times m}$ are the coefficient matrices for estimation and to be identified. Let $n = mn_a + rn_b + mn_c$, designate the system parameter vectors

$$\theta := [A_1, A_2, ..., A_{n_A}, B_1, B_2, ..., B_{n_b}] \in R^{m\times(mn_a+rn_b)},$$

$$\Theta = [C_1, C_2, ..., C_{n_c}] \in \mathbb{R}^{m\times(mn_c)},$$

$$\vartheta := [\theta, C_1, C_2, ..., c_{n_c}] \in R^{m\times n},$$

$$\vartheta := [\theta^T, \Theta^T]^T \in R^{m\times n},$$

$$n := mn_a + rn_b + mn_c$$

and the corresponding information vectors

$$\varphi(t) := [-y(t-1), -y(t-2), ..., -y(t-n_a), u(t-1), u(t-2), ..., u(t-n_b)] \in \mathbb{R}^{(mn_a+rn_b)},$$

$$\phi(t) := [\nu(t-1), \nu(t-2), ..., \nu(t-n_c)] \in \mathbb{R}^{mn_c},$$

$$\psi(t) := [\varphi(t), \nu(t-1), \nu(t-2), ..., \nu(t-n_c)] \in \mathbb{R}^n,$$

$$\psi(t) = [\varphi(t), \phi] \in \mathbb{R}^n.$$

Based on the mentioned definitions and equations (1), we attain the following identification model

$$
\begin{aligned}
y(t) =&[I - A(z)]y(t) + B(z)u(t) + C(z)\nu(t)\\
=&(-A_1z^{-1} - A_2z^{-2} - ... - A_{n_a}z^{-n_a})y(t) + (B_1z^{-1} + B_2z^{-2} + ... + B_{n_b}z^{-n_b})u(t)\\
&+ (I + C_1z^{-1} + C_2z^{-2} + ... + C_{n_c}z^{-n_c})\nu(t)\\
=&- A_1y(t-1) - A_2y(t-2) - ... - A_{n_a}y(t-n_a)\\
&+ B_1u(t-1) + B_2u(t-2) + ... + B_{n_b}u(t-n_b)\\
&+ \nu(t) + C_1\nu(t-1) + C_2\nu(t-2) + ... + C_{n_c}\nu(t-n_c)\\
=&\theta[-y(t-1) - y(t-2) - ... - y(t-n_a) + u(t-1) + u(t-2) + ... + u(t-n_b)]\\
&+ \Theta[\nu(t-1) + \nu(t-1) + ... + \nu(t-n_c)] + \nu(t)\\
=&\vartheta[-y(t-1) - y(t-2) - ... - y(t-n_a) + u(t-1) + u(t-2) + ... + u(t-n_b)\\
&+ \nu(t-1) + \nu(t-1) + ... + \nu(t-n_c)] + \nu(t)\\
y(t) =&\varphi(t)\theta + \phi(t)\Theta + \nu(t) \qquad (2)\\
y(t) =&\vartheta\psi(t) + \nu(t). \qquad (3)
\end{aligned}
$$

Now we take L as data length ($L \gg n$) and assume the stacked output matrix as Y, the stacked information matrices Φ and Ψ and the stacked noise matrix as V.

$$
\begin{aligned}
Y =&[y(1), y(2), ..., y(L)] \in \mathbb{R}^{m\times L}\\
\Phi =&[\varphi(1), \varphi(2), ..., \varphi(L)] \in \mathbb{R}^{(mn_a+rn_b)\times L}\\
\zeta =&[\phi(1), \phi(2), ..., \phi(L)] \in \mathbb{R}^{mn_c\times L}\\
\Psi =&[\psi(1), \psi(2), ..., \psi(L)] \in \mathbb{R}^{n\times L}\\
V =&[\nu(1), \nu(2), ..., \nu(L)] \in \mathbb{R}^{m\times L}.
\end{aligned}
$$

The ultimate aim of this paper is done in the next section which identifies the ϑ parameter using parameter estimation algorithms.

3 Mathematics of two gradient-based iterative algorithms

3.1 Gradient-based iterative algorithm

Considering Eq. (3), we yield the following equation

$$Y = \vartheta\Psi + V. \qquad (4)$$

Now we take a criterion function of a quadratic form as follows:

$$J = \|Y - \vartheta\Psi\|. \qquad (5)$$

Our aim here is to minimize $J(\vartheta)$ by using the search of negative gradient, so $\hat{\vartheta}_k$ can be calculated which is the estimation of ϑ at iteration k. The following result will be derived using Eq. (5)

$$\hat{\vartheta}_k = \hat{\vartheta}_{k-1} + \mu_k[Y - \hat{\vartheta}\Psi]\Psi^T. \qquad (6)$$

where μ is the convergence factor or iterative step size. The only problem we face here is that the information vector $\psi(t)$ in Ψ includes unknown parameters of noise $\nu(t-i)$. Regarding this issue, Eq. (6) can not provide the estimation of $\hat{\vartheta}_k$. To tackle this problem, capitalization is done on the corresponding estimate of noise terms as $\hat{\nu}_{k-1}$.

$$
\begin{aligned}
\hat{\psi}_k(t) :=&[\varphi(t), \hat{\nu}_{k-1}(t-1), \hat{\nu}_{k-1}(t-2), ..., \hat{\nu}_{k-1}(t-n_c)] \in \mathbb{R}^n\\
\hat{\Psi}_k =&[\hat{\psi}_k(1), \hat{\psi}_k(2), ..., \hat{\psi}_k(L)] \in \mathbb{R}^{n\times L}
\end{aligned}
$$

From Eq. (3)

$$\nu(t) = y(t) - \vartheta\psi(t) \tag{7}$$

and by replacing parameters with their estimation, we reach the following equations

$$\begin{aligned} \hat{\nu}(t) &= y(t) - \hat{\vartheta}\hat{\psi}(t) \\ \hat{\vartheta}_k &= \hat{\vartheta}_{k-1} + \mu_k[Y - \hat{\vartheta}_{k-1}\hat{\Psi}_k]\hat{\Psi}_k^T. \end{aligned} \tag{8}$$

So as to make sure that $\hat{\vartheta}_k$ converge, we assume a conservative choice of μ_k

$$0 < \mu_k \leq \frac{2}{\lambda_{max}[\hat{\Psi}_k\hat{\Psi}_k^T]} \tag{9}$$

and $\lambda_{max}[X]$ symbolizes the highest eigenvalue of the square matrix X. Based on the provided equations, the GI algorithm for the CARMA system can be derived as follows

$$\hat{\vartheta}_k = \hat{\vartheta}_{k-1} + \mu_k[Y - \hat{\vartheta}_{k-1}\hat{\Psi}_k]\hat{\Psi}_k^T, \tag{10}$$

$$\mu_k = \frac{2}{\lambda_{max}[\hat{\Psi}_k\hat{\Psi}_k^T]}, \tag{11}$$

$$\hat{\Psi}_k = [\hat{\psi}_k(1), \hat{\psi}_k(2), ..., \hat{\psi}_k(L)] \in \mathbb{R}^{n\times L}, \tag{12}$$

$$Y = [y(1), y(2), ..., y(L)] \in \mathbb{R}^{m\times L}, \tag{13}$$

$$\varphi(t) = [-y(t-1), -y(t-2), ..., -y(t-n_a), u(t-1), u(t-2), ..., u(t-n_b)]^T \in \mathbb{R}^{(mn_a+rn_b)} \tag{14}$$

$$\hat{\psi}(t) := [\varphi(t), \hat{\nu}(t-1), \hat{\nu}(t-2), ..., \hat{\nu}(t-n_c)] \in \mathbb{R}^n, \tag{15}$$

$$\hat{\theta}_k = \hat{\vartheta}_k(1 : mn_a + mn_b, 1 : m), \tag{16}$$

$$\hat{\nu}(t) = y(t) - \hat{\vartheta}\hat{\psi}(t). \tag{17}$$

The step-by-step GI algorithm,which is defined in Eqs. (10-17), and aims to calculate the estimation of $\hat{\vartheta}(t)$ for CARMA system, is depicted in Fig. 2 and is brought as follow:

1. Gather the input-output data $u(t)$, $y(t)$ for $t = 1, 2, ..., L$ ($L \gg n$) and make the stacked output matrix Y by Eq. (13) and $\varphi(t)$ by Eq. (14),
2. Take $k = 1$ as iteration variable and fix other primary values: $\hat{\vartheta}_0 = \frac{1}{P_0}$ for $p_0 = 10^6$ and $\hat{\nu}_0(t)$ as a random variable and set some small preset ε,
3. Form $\hat{\psi}_k(t)$ by Eq. (15) and $\hat{\Psi}_k$ by Eq. (12),
4. Construct μ_k by Eq. (11) and update estimation of $\hat{\vartheta}_k$ by Eq. (10),
5. set $\hat{\theta}_k$ by Eq. (16),
6. Calculate $\hat{\nu}_k(t) =$ by Eq. (17),
7. Contrast $\hat{\vartheta}_k$ with $\hat{\vartheta}_{k-1}$, if $\|\hat{\vartheta}_k - \hat{\vartheta}_{k-1}\| \leq \varepsilon$ then end the algorithm and get the iteration value k and estimation of $\hat{\vartheta}_k$, in all other respects extend k by 1 and start from step 3.

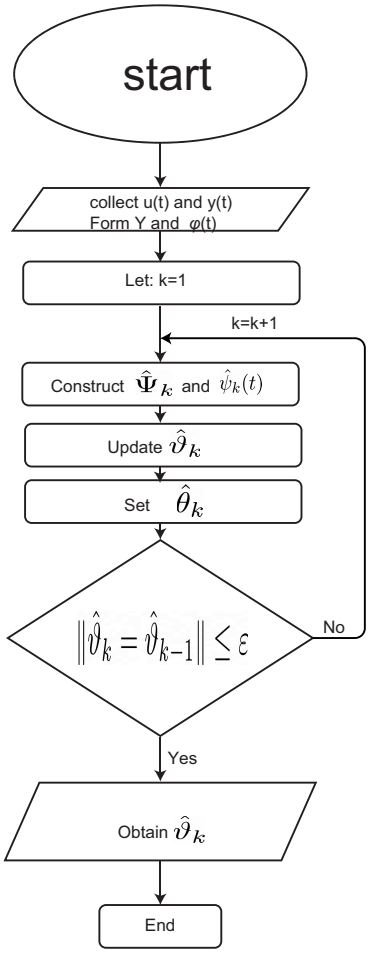

Figure 2: Flowchart of GI algorithm

3.2 Two-stage gradient-based iterative algorithm

In this section, 2S-GI is derived. For gaining a two-stage algorithm, we first consider two intermediate variables defined as

$$y_1(t) := y(t) - \Theta\phi(t), \tag{18}$$

$$y_2(t) := y(t) - \theta\varphi(t). \tag{19}$$

Now from Eq. (2), two fictitious subsystems can be derived as

$$y_1(t) = \theta\varphi(t) + \nu(t) \tag{20}$$

$$y_2(t) = \Theta\phi(t) + \nu(t) \tag{21}$$

Take these matrices into consideration

$$Y_1 = [y_1(1), y_1(2), ..., y_1(L)] \in \mathbb{R}^{m\times L},$$

$$Y_2 = [y_2(1), y_2(2), ..., y_2(L)] \in \mathbb{R}^{m\times L}$$

which are two stacked output vectors. Then these two equations can be derived

$$Y_1 = Y - \Theta\zeta \tag{22}$$

$$Y_2 = Y - \theta\Phi. \tag{23}$$

From Eqs. (24-25), we reach the following equations

$$Y_1 = \theta\Phi + V$$

$$Y_2 = \Theta\zeta + V.$$

Now two quadratic criterion function is attained

$$J_1(\theta) = \|Y_1 - \theta\Phi\|^2, \tag{24}$$

$$J_2(\Theta) = \|Y_2 - \Theta\zeta\|^2. \tag{25}$$

Similar to the previous section, our aim here is to minimize $J_1(\theta)$ and $J_2(\Theta)$ by using the search of negative gradient, so $\hat{\theta}_k$ and $\hat{\Theta}_k$ which are the estimation of θ and Θ at iteration k respectively can be calculated. Here k is the iteration variable. Based on (24) and (25), we have the following equations:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \mu_{k1}[Y_1 - \hat{\theta}\Phi]\Phi^T \tag{26}$$

$$\hat{\Theta}_k = \hat{\Theta}_{k-1} + \mu_{k2}[Y_2 - \hat{\Theta}\zeta]\zeta^T. \tag{27}$$

The only problem we face here is that the information vector ϕ in ζ includes unknown noise parameters $\nu(t-i)$, so (27) cannot provide the estimation of $\hat{\Theta}_k$. To tackle this problem, we capitalize on the corresponding estimate of noise terms as $\hat{\nu}_{k-1}$.

$$\hat{\phi}_k(t) := [\hat{\nu}_{k-1}(t-1), \hat{\nu}_{k-1}(t-2), ..., \hat{\nu}_{k-1}(t-n_c)] \in \mathbb{R}^{mn_c}$$

$$\hat{\zeta}_k = [\hat{\phi}_k(1), \hat{\phi}_k(2), ..., \hat{\phi}_k(L)] \in \mathbb{R}^{mn_c \times L}.$$

From Eq. (2), following expression will be yield

$$\hat{\nu}(t) = y(t) - \hat{\theta}\varphi(t) - \hat{\Theta}\hat{\phi}(t). \tag{28}$$

So as to make sure that $\hat{\theta}_k$ and $\hat{\Theta}_k$ converge, we assume a conservative choice of μ_{k1} and μ_{k2}

$$0 < \mu_{k1} \leq \frac{2}{\lambda_{max}[\hat{\Phi}_k\hat{\Phi}_k^T]}, \tag{29}$$

$$0 < \mu_{k2} \leq \frac{2}{\lambda_{max}[\hat{\zeta}_k\hat{\zeta}_k^T]} \tag{30}$$

and $\lambda_{max}[X]$ symbolize the highest eigenvalue of the square matrix X. Based on the brought equations, the 2S-GI algorithm for the CARMA system is

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \mu_{k1}[Y_1 - \hat{\theta}\Phi]\Phi^T, \tag{31}$$

$$\hat{\Theta}_k = \hat{\Theta}_{k-1} + \mu_{k2}[Y_2 - \hat{\Theta}\hat{\zeta}]\hat{\zeta}^T, \tag{32}$$

$$\mu_{k1} = \frac{2}{\lambda_{max1}[\hat{\Phi}_k\hat{\Phi}_k^T]}, \tag{33}$$

$$\mu_{k2} = \frac{2}{\lambda_{max}[\hat{\zeta}_k\hat{\zeta}_k^T]}, \tag{34}$$

$$\Phi = [\varphi(1), \varphi(2), ..., \varphi(L)] \in \mathbb{R}^{(mn_a+rn_b)\times L}, \tag{35}$$

$$\hat{\zeta} = [\hat{\phi}(1), \hat{\phi}(2), ..., \hat{\phi}(L)] \in \mathbb{R}^{mn_c\times L}, \tag{36}$$

$$Y = [y(1), y(2), ..., y(L)] \in \mathbb{R}^{m\times L}, \tag{37}$$

$$\varphi(t) := [-y(t-1), -y(t-2), ..., -y(t-n_a), u(t-1), u(t-2), ..., u(t-n_b)]^T \in \mathbb{R}^{(mn_a+rn_b)}, \tag{38}$$

$$\hat{\phi}_k(t) := [\hat{\nu}_{k-1}(t-1), \hat{\nu}_{k-1}(t-2), ..., \hat{\nu}_{k-1}(t-n_c)] \in \mathbb{R}^{mn_c}, \tag{39}$$

$$\hat{\nu}(t) = y(t) - \hat{\theta}\varphi(t) - \hat{\Theta}\hat{\phi}(t). \tag{40}$$

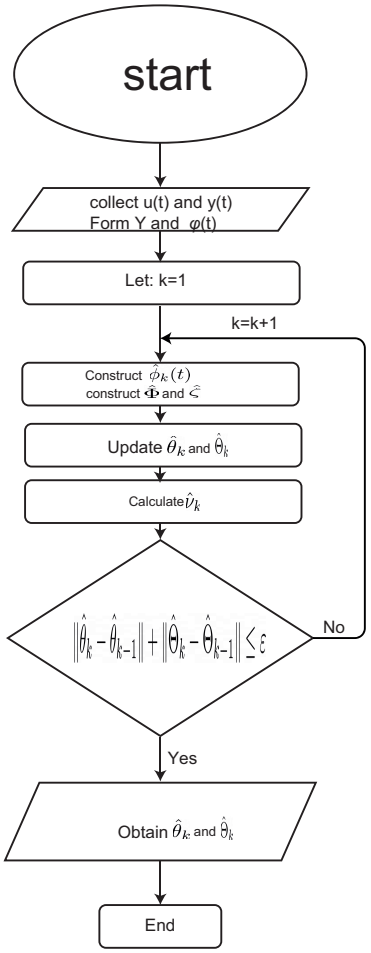

Figure 3: Flowchart of 2S-GI algorithm

The step-by-step 2S-GI algorithm, which is defined in Eqs.(31-40), and aims to calculate the estimation of $\hat{\theta}(t)$ and $\hat{\Theta}(t)$ for CARMA system, is depicted in Fig. 3 and is brought as follow:

1. Gather the input-output data $u(t)$, $y(t)$ for $t = 1, 2, ..., L$ ($L \gg n$) and make the stacked output matrix Y by (37) and $\varphi(t)$ by (38),

2. Take $k = 1$ as iteration variable and fix other primary values: $\hat{\theta}_0 = \frac{1}{P_0}$ and $\hat{\Theta}_0 = \frac{1}{P_0}$ for $p_0 = 10^6$ and $\hat{\nu}_0(t)$ as a random variable and set some small preset ε,
3. Form $\hat{\phi}_k(t)$ by Eq. (39), Φ and $\hat{\zeta}$ by Eq. (35) and Eq. (36) respectively,
4. Construct μ_{k1} and μ_{k2} by Eq. (33) and Eq. (34). Also update the estimate $\hat{\theta}_k$ and $\hat{\Theta}_k$ by Eq. (31) and Eq. (32), respectively,
5. Calculate $\hat{\nu}_k(t) =$ by Eq. (40),
6. If $\|\hat{\theta}_k - \hat{\theta}_{k-1}\| + \|\hat{\Theta}_k - \hat{\Theta}_{k-1}\| \leq \varepsilon$, attain the parameter estimate $\hat{\theta}$ and $\hat{\Theta}$, in all other respects extend k by 1 and restart from step 3.

4 Tumor model

Here we introduce a tumor model which was presented in Lobato et al. (2016).

$$\begin{aligned}
\dot{N}(t) &= r_2 N(t)(1 - b_2 N(t)) - c_4 T(t) N(t) - a_3 u(t), & N(0) &= N_o \\
\dot{T}(t) &= r_1 T(t)(1 - b_1 T(t)) - c_2 I(t) T(t) - c_3 T(t) N(t) - a_2 u(t), & T(0) &= T_o \\
\dot{I}(t) &= s + \frac{\rho I(t) T(t)}{\alpha + T(t)} - c_1 I(t) T(t) - d_1 I(t) - a_1 u(t), & I(0) &= I_o
\end{aligned}$$

in which the represented number of immune cells at time t is denoted with I, T denotes the number of tumor cells at time t, N symbolizes the amount of normal (host) cells at time t and finally, the control strategy is u. Parameters and their values for the simulation section is presented in Table 1.

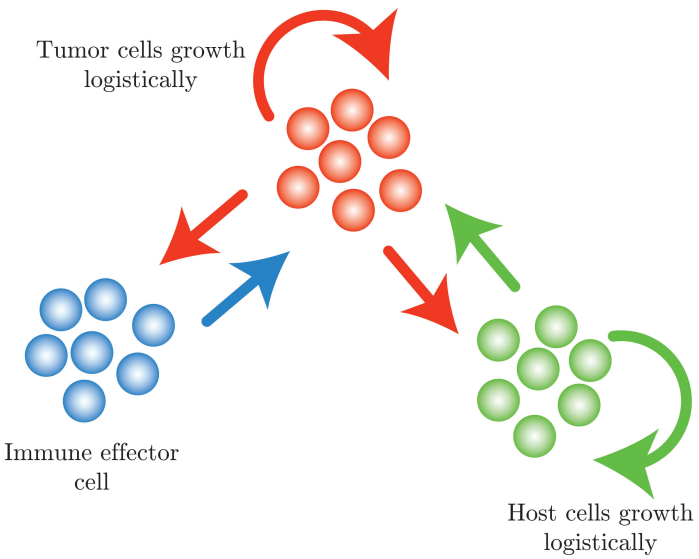

Figure 4: T-I-N interaction

Table 1: Parameters and their values

parameters	values	parameters	values
a_1	0.2	a_2	0.3
a_3	0.1	b_1	1
b_2	1	α	0.3
c_1	1	c_2	0.5
c_3	1	c_4	1
d_1	0.2	ρ	0.01
r_1	1.5	r_2	1
s	0.33		

5 Simulation

In this section, a tumor model is illustrated in a polynomial model and afterward, the parameters of the model are estimated through figures and tables. The tumor polynomial model for CARMA modeling is

$$A_1(z) = \begin{bmatrix} 0.0152 & 0.0186 & 0.0243 \\ 0.0217 & 0.0230 & 0.0245 \\ 0.0201 & 0.0185 & 0.0196 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}. \tag{41}$$

$$B_1(z) = \begin{bmatrix} -0.0004 & 0 & 0 \\ 0.0002 & 0 & 0 \\ 0.0005 & 0 & 0 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}. \tag{42}$$

$$C_1(z) = \begin{bmatrix} 0.0049 & 0 & 0 \\ 0 & 0.0043 & 0 \\ 0 & 0 & 0.0016 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}. \tag{43}$$

It is noteworthy to say that parameters that are zero will not be identified in the subsequent tables because they remain constant throughout the identification process.

Table 2: Estimation results for $\sigma^2 = (0.1)^2$

Approach	$t = L$	a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}
	500	0.0097	0.0102	0.0097	0.0099	0.0099	0.0100
GI	1000	0.0103	0.0105	0.0098	0.0098	0.0096	0.0099
	2000	0.0101	0.0099	0.0100	0.0102	0.0098	0.0097
True Value		0.0152	0.0186	0.0243	0.0217	0.0230	0.0245

Approach	t=L	a_{31}	a_{32}	a_{33}	b_{11}	b_{21}	b_{31}
	500	0.0103	0.0095	0.0102	0.0100	0.0092	0.0099
GI	1000	0.0100	0.0103	0.0102	0.0053	0.0061	-0.0026
	2000	0.0102	0.0104	0.0102	-0.0009	0.0059	-0.0008
True Value		0.0201	0.0185	0.0196	-0.0001	-0.0001	0.0000

Approach	t=L	c_{11}	c_{22}	c_{33}	$\delta(\%)$
	500	0.0050	0.0006	0.0003	35.3257
GI	1000	0.0055	-0.0016	-0.0007	30.6193
	2000	0.0087	-0.0014	-0.0017	29.7729
True Value		0.0049	0.0043	0.0016	

Table 3: Estimation results for $\sigma^2 = (0.1)^2$

Approach	$t = L$	a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}
2S-GI	500	0.0185	0.0179	0.0242	0.0229	0.0232	0.0276
	1000	0.0174	0.0186	0.0190	0.0212	0.0242	0.0179
	2000	0.0197	0.0162	0.0188	0.0181	0.0193	0.0219
True Value		0.0152	0.0186	0.0243	0.0217	0.0230	0.0245

Approach	t=L	a_{31}	a_{32}	a_{33}	b_{11}	b_{21}	b_{31}
2S-GI	500	0.0182	0.0219	0.0211	0.0000	-0.0003	-0.0006
	1000	0.0215	0.0202	0.0127	0.0003	0.0004	0.0001
	2000	0.0183	0.0182	0.0183	-0.0001	-0.0001	0.0000
True Value		0.0201	0.0185	0.0196	-0.0001	-0.0001	0.0000

Approach	t=L	c_{11}	c_{22}	c_{33}	$\delta(\%)$
2S-GI	500	-0.0073	-0.0071	-0.0071	10.0173
	1000	0.0057	0.0055	0.0054	3.7441
	2000	0.0035	0.0032	0.0035	2.5776
True Value		0.0049	0.0043	0.0016	

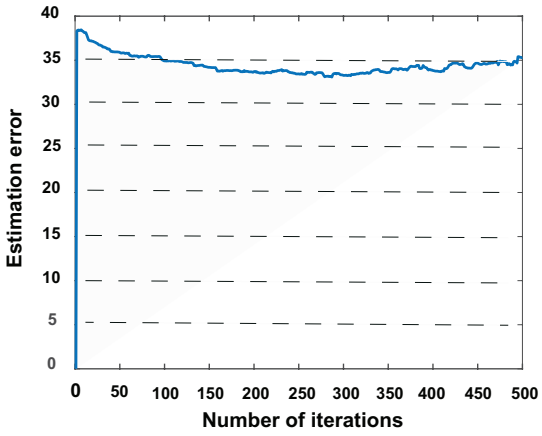

Figure 5: Estimation error of GI approach for $L = 500$ and $\sigma^2 = 0.1^2$

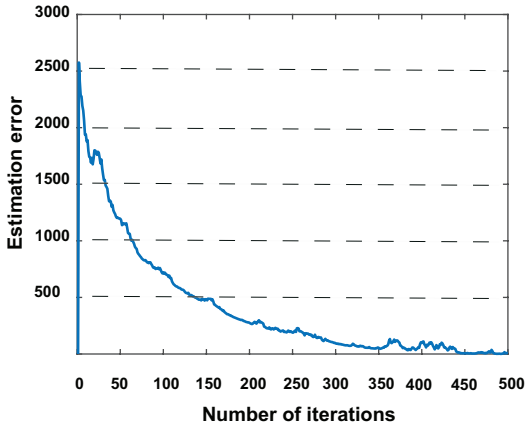

Figure 6: Estimation error of 2S-GI approach for $L = 500$ and $\sigma^2 = 0.1^2$

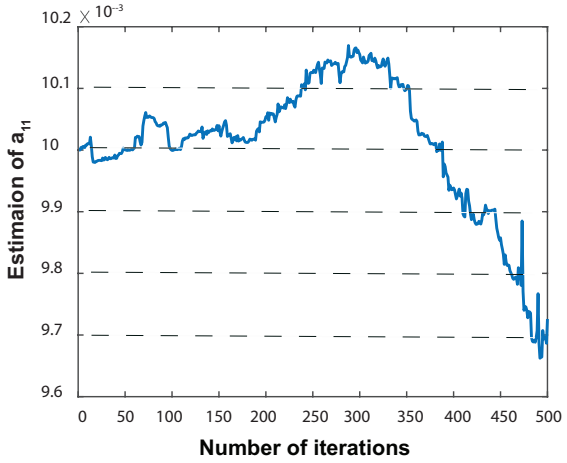

Figure 7: Estimation of a_{11} with GI approach for $L = 500$ and $\sigma^2 = 0.1^2$

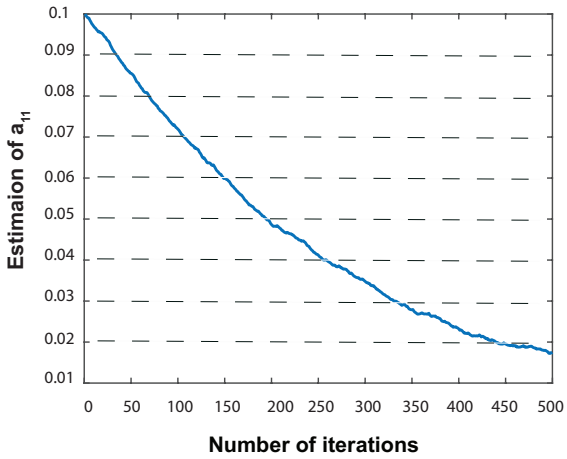

Figure 8: Estimation of a_{11} with 2S-GI approach for $L = 500$ and $\sigma^2 = 0.1^2$

6 Discussion and results

To tackle the problem of tumors in the body of patients, tumor cells must be destroyed with minimum effect on normal cells. Also, a tumor's response is reliant on several parameters, namely the severity of the ailment, the efficacy of the treatment, and the strength of the patient's own immune response. The main goal here is to look for the most efficient protocol for drug administration. The control strategy $u(t)$ is through chemotherapy, immunotherapy, or a combination of both treatments. Therefore having a mathematical model and controlling it is a necessity. For making the controlling process easier, we represented a polynomial model and by GI and 2S-GI algorithms we estimated the parameter of the tumor model. This parameter estimation makes the pathway for controlling the tumor system. Afterward, by having a suitable model for tumors and a control strategy $u(t)$, control engineers can take advantage of optimal control, adaptive control, or other means to solve the problem of tumor cell growth within the body and make cancer treatment happen.

From table 2 and 3, it is conceived that as the number of data increases error in percent reduces for both GI and 2S-GI methods. Also, it is clear that the 2S-GI method can decrease the amount of error more significantly than the GI method. For instance, GI produces amounts

between 29% and 35% for $L = 500, 1000, 2000$ but 2S-GI gives rise to amounts between 2% and 10% for the same amounts of data.

Figure 5 shows that the tumor model parameter estimation for the number of data 500 and amount of noise 0.1 reaches its minimum after 300 iterations to approximately 33 but then increases marginally to 35 at the end of the span of iterations. This form of convergence is not suitable for a parameter estimation approach and it should have a declining trend throughout the whole span of iterations. This small fluctuation can lead to bad identification results. Whereas figure 6 with the same conditions, for the 2S-GI approach shows an overall decreasing trend and it converges to a less amount of error in contrast to GI and it is reaching its nadir which is nearly zero, after only 450 iterations and then becomes stable. Therefore 2S-GI method for the presented tumor model benchmark can produce significantly less amount of error though with a slower rate of convergence.

From figure 7 and 8, it can be comprehended that estimation of a_{11} goes through some fluctuation with GI algorithm while 2S-GI algorithm converges in a better way. Therefore 2S-GI has a better reaction than GI algorithm while converging to its final point and it is concluded that 2S-GI has a better performance in the convergence of dynamics of the tumor model system.

Overall, from figures 5, 6 and tables 2, 3, we can derive these results:

- 2S-GI approach error convergences to zero or its final point much better than GI approach.
- 2S-GI produces less error than GI.
- As the number of iterations increases, the amount of estimation errors reduces in a clear way and parameters estimation converges in a much better way.

7 Conclusion

In this contribution, mathematical theories and algorithms of two identification methods, i.e., GI and 2S-GI for MIMO CARMA systems, were developed in which 2S-GI is the main novelty. Furthermore, a tumor model with one input and three outputs was presented. By means of introduced parameter estimation approaches, the model was identified and its parameters were estimated. Also, the GI and 2S-GI algorithms indicated that they both are able to estimate parameters of a polynomial CARMA configuration at a fast convergence rate producing an insignificant amount of error. For future works it is highly suggested that scholars pay their attention to deriving a 2S-GI approach for the CARARMA model of MIMO systems, especially for the benchmark of tumor models. Also controlling the presented polynomial model by means of adaptive control or other types of controllers is on demand.

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