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ORIGINAL ARTICLE

Individual Differences in Mathematical Problem-Solving Skills Among 3- to 5-Year-Old Preschoolers

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Abstract

The aim of this study was to investigate individual differences in mathematical problem-solving among 3- to 5-year-old children ($N=328$; $n_{3\text{-year-olds}}=115$, $n_{4\text{-year-olds}}=167$, $n_{5\text{-year-olds}}=46$). First, we examined the extent to which children in this age group were able to solve open and closed non-routine mathematical problems representing a variety of mathematical domains. Second, we investigated the extent to which underlying academic and cognitive skills (i.e., expressive and receptive language, visuospatial, and early numeracy skills) were associated with individual differences in mathematical problem-solving concurrently and longitudinally (i.e., one year later). The results showed that 4- to 5-year-olds were able to solve a variety of non-routine mathematical problems. However, though 3-year-olds were also able to solve a variety of problems, the mathematical problem-solving measure did not meet the reliability criteria, resulting in excluding 3-year-olds from further analyses. Expressive and receptive language, visuospatial, and early numeracy skills were associated with mathematical problem-solving concurrently among 4-year-olds. Among 5-year-olds, only visuospatial and early numeracy skills were associated with mathematical problem-solving. Furthermore, only prior mathematical problem-solving skills and early numeracy skills predicted mathematical problem-solving skills longitudinally. These findings indicate that preschoolers are able to solve open and closed non-routine mathematical problems representing a variety of mathematical domains. Additionally, individual differences may stem not only from differences in mathematical problem-solving skills but also from early numeracy.

Keywords Early childhood education · Early numeracy · Individual differences · Mathematical problem-solving · Preschoolers

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Résumé
L’objectif de cette étude était d’examiner les différences individuelles dans la résolution de problèmes mathématiques chez les enfants de 3 à 5 ans ($N = 328$; $n_{3\ \text{ans}} = 115$, $n_{4\ \text{ans}} = 167$, $n_{5\ \text{ans}} = 46$). Tout d’abord, nous avons examiné dans quelle mesure les enfants de ce groupe d’âge étaient capables de résoudre des problèmes mathématiques non routiniers ouverts et fermés représentant une variété de domaines mathématiques. Deuxièmement, nous avons étudié dans quelle mesure les aptitudes scolaires et cognitives sous-jacentes (c’est-à-dire le langage expressif et réceptif, les aptitudes visuospatiales et les premières aptitudes au calcul) étaient associées aux différences individuelles en matière de résolution de problèmes mathématiques de manière simultanée et longitudinale (c’est-à-dire un an plus tard). Les résultats ont montré que les enfants de 4 à 5 ans étaient capables de résoudre une variété de problèmes mathématiques non routiniers. Cependant, bien que les enfants de 3 ans soient également capables de résoudre une variété de problèmes, la mesure de la résolution de problèmes mathématiques n’a pas satisfait aux critères de fiabilité, ce qui a conduit à exclure les enfants de 3 ans des analyses ultérieures. Les compétences linguistiques expressives et réceptives, visuospatiales et de calcul précoce ont été associées à la résolution de problèmes mathématiques simultanément chez les enfants de 4 ans. Chez les enfants de 5 ans, seules les compétences visuospatiales et de calcul précoce ont été associées à la résolution de problèmes mathématiques. En outre, seules les aptitudes antérieures à la résolution de problèmes mathématiques et les aptitudes précoces à la numératie permettaient de prédire les aptitudes à la résolution de problèmes mathématiques sur une base longitudinale. Ces résultats indiquent que les enfants d’âge préscolaire sont capables de résoudre des problèmes mathématiques ouverts et fermés non routiniers représentant une variété de domaines mathématiques. Par ailleurs, les différences individuelles peuvent découler non seulement de différences dans les aptitudes à la résolution de problèmes mathématiques, mais aussi de différences dans les aptitudes précoces à la numération.

Resumen
El objetivo de este estudio era investigar las diferencias individuales en la resolución de problemas matemáticos entre niños de 3 a 5 años ($N = 328$; $n_{3\ \text{años}} = 115$, $n_{4\ \text{años}} = 167$, $n_{5\ \text{años}} = 46$). En primer lugar, examinamos hasta qué punto los niños de este grupo de edad eran capaces de resolver problemas matemáticos abiertos y cerrados no rutinarios que representaban una variedad de dominios matemáticos. En segundo lugar, investigamos hasta qué punto las habilidades académicas y cognitivas subyacentes (es decir, el lenguaje expresivo y receptivo, las habilidades visuoespaciales y las habilidades numéricas tempranas) se asociaban con diferencias individuales en la resolución de problemas matemáticos de forma concurrente y longitudinal (es decir, un año después). Los resultados mostraron que los niños de 4 a 5 años eran capaces de resolver diversos problemas matemáticos no rutinarios. Sin embargo, aunque los niños de 3 años también eran capaces de resolver diversos problemas, la medida de resolución de problemas matemáticos no cumplía los criterios de fiabilidad, por lo que se excluyó a los niños de 3 años de los análisis posteriores. El lenguaje expresivo y receptivo, las habilidades visuoespaciales y numéricas tempranas se asociaron con

la resolución de problemas matemáticos de forma concurrente entre los niños de 4 años, mientras que entre los niños de 5 años, sólo las habilidades visuoespaciales y numéricas tempranas se asociaron con la resolución de problemas matemáticos. Además, sólo las habilidades previas de resolución de problemas matemáticos y las habilidades numéricas tempranas predijeron longitudinalmente las habilidades de resolución de problemas matemáticos. Estos resultados indican que los preescolares son capaces de resolver problemas matemáticos abiertos y cerrados no rutinarios que representan una variedad de dominios matemáticos. Además, las diferencias individuales pueden derivarse no sólo de las diferencias en las habilidades de resolución de problemas matemáticos, sino también de la numéricas temprana.

Introduction

Mathematical problem-solving skills serve as an important building block for learning (Dewolf et al., 2014; Fuchs et al., 2012) as these skills require children to combine and apply mathematical knowledge in a new way in order to solve a previously unknown problem (Verschaffel et al., 2000). Due to their applied nature, mathematical problem-solving skills are essential for later success in not only mathematics but also in everyday life and future working life (OECD, 2017). A vast majority of research in mathematical problem-solving has concentrated on school-aged children, while research concerning preschoolers' mathematical problem-solving skills has been largely overlooked. Preschoolers have been shown to be able to solve tasks involving arithmetic operations that can be solved by using a familiar strategy (Hornburg et al., 2018; Kyttälä et al., 2014; Martin et al., 2014). A few studies have also indicated that young children may be able to solve more demanding mathematical problem-solving tasks that require combining and applying multiple arithmetic operations (Carpenter et al., 1993). However, it remains unclear to what extent 3- to 5-year-old preschoolers are able to solve mathematical problems representing a variety of mathematical domains. Though several studies have shown that cognitive and academic skills (e.g., expressive and receptive language, visuospatial, and early numeracy skills) are crucial for solving a mathematical problems among older than preschool-aged children, less is known concerning the development of preschool-aged children's mathematical problem-solving skills (Lin, 2020). Considering the cumulative nature of learning mathematics and the need of fostering skills that are required throughout children's lives, it is important that more research-based knowledge is produced concerning young children's mathematical problem-solving skills. Such investigation will result in a more profound understanding of young children's mathematical knowledge, which will in turn contribute to developing suitable assessment tools to measure a variety of preschool-aged children's mathematical problem-solving skills and designing effective instruction in early childhood education. Consequently, the aim of this study was to investigate individual differences in mathematical problem-solving skills among 3- to 5-year-old preschoolers.

Mathematical Problem-Solving

Mathematical problem-solving requires a child to combine and apply mathematical knowledge in a new way in order to solve a previously unknown problem (Schoenfeld, 1985; Verschaffel et al., 2000). Mathematical problem-solving encompasses a variety of tasks presented in different formats and contexts. Thus, prior research has utilized many different terms of the same tasks (e.g., mathematical problems, word problems or story problems). For clarity reasons, we refer to mathematical problem-solving in this study (Schoenfeld, 1985).

One way of classifying mathematical problem-solving tasks is between the dimensions of routine and non-routine mathematical problem-solving (Daroczy et al., 2015; Laine et al., 2018). Routine tasks are characterized as tasks that can be solved following a specific step-by-step procedure which is usually familiar to the problem solver (Polya, 1957). Routine tasks also include all the necessary information to solve the task. One example of a routine task is the following: 'If Mandy has three candies and mom gives her two more, how many candies does she now have?' (Strohmaier et al., 2021). As the solution process is often evident for the solver, routine tasks do not fill the definition of having to combine and apply mathematical knowledge in a new way. Non-routine tasks on the other hand can be described as more problematic than routine tasks. Non-routine problems may require children to incorporate real-world knowledge or combine multiple mathematical operations for deducing the answer. An example of a non-routine problem is: 'The distance between two poles is 12 m. How many 1.5 m long pieces of rope should one tie together to connect both poles?' (DeWolf et al., 2014; Verschaffel et al., 1994). In this example, children need to understand the problem situation in the real world to acknowledge that tying knots and going around the poles will take some of the length of the rope which makes the problem more complex. Notably, whether a task can be defined as a routine or a non-routine is influenced by the child's skills (Daroczy et al., 2015). Tasks that could be defined as non-routine among preschool-aged children may be routine tasks to older children who naturally have more developed cognitive skills and more experience with real-world situations and mathematical problems in the school context.

In addition to being a routine or a non-routine problem-solving task, problems can also be open or closed (Laine et al., 2018). Closed problems traditionally have a predefined starting point and only one correct solution. In the previous example of Mandy having candies, there is only one correct answer, which can be deduced by solving an arithmetic operation and all necessary information needed to solve the task is given. Therefore, this task is categorized as a closed and routine task. In contrast to closed problems, open problems can have an open starting point or multiple possible solutions, even an infinite amount (Daroczy et al., 2015; Laine et al., 2018). In the example of a non-routine problem of a man with the rope, the answer is dependent on how much rope is spent for the knots which is why this task does not have one sole answer. Therefore, this task is an example of an open and non-routine task. In the real world, problems are hardly as straightforward as closed routine tasks, highlighting the importance of open and non-routine problem-solving skills (DeWolf et al., 2014; Strohmaier et al., 2021).

Preschooler's Mathematical Problem-Solving Skills

Young children start developing mathematical knowledge before the preschool years (Carpenter et al., 1993; Kyttälä et al., 2014). In fact, young children are naturally curious and through play, they often find different problems to solve (Charlesworth & Leali, 2012; Lopes et al., 2017). Prior research has well established that for example, infants' ability to recognize numerosities (i.e., subitizing) and differences between numerosities (i.e., approximate number system) are important for developing more advanced mathematical knowledge such as performing arithmetic operations in the preschool years (Clements & Sarama, 2021; Dehaene, 2011). However, less is known about preschool-aged children's ability to solve mathematical problems and the skills affecting such abilities.

The few studies conducted among young children have shown that children already between the ages of 3 to 7 years are able to solve closed routine mathematical tasks that involve arithmetic operations such as 'If Laura has two stickers and her mom gives her two more, how many stickers does she now have?' (Hornburg et al., 2018; Kyttälä et al., 2014; Martin et al., 2014). However, as noted previously, problems in which children need to apply mathematical knowledge in the real world are hardly routine and require only arithmetic operations. Carpenter et al. (1993) showed that more than 50% of 5- to 6-year-old kindergartners included in their sample were able to correctly solve a closed problem requiring multiple steps in arithmetic operations (i.e., 'Maggie had 3 packages of cupcakes. There were 4 cupcakes in each package. She ate 5 cupcakes. How many are left?'). In addition, half of the kindergartners were able to correctly solve a closed non-routine problem requiring arithmetics (i.e., '19 children are taking a mini bus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has 7 seats. How many children will have to sit three to a seat, and how many can sit two to a seat?'; Carpenter et al., 1993). The study of Matalliotaki (2012) also showed that around 30–40% of 5- to 6-year-olds were able to correctly solve a closed problem involving division (i.e., 'How many children can one equip with these socks?') when the problem was accompanied with a representational picture. As the line of research involving young children's non-routine mathematical problem-solving has been limited to closed arithmetic problems and kindergartners, it remains unresolved whether children between the ages of 3–5 are able to solve open and closed non-routine mathematical problems of a variety of mathematical domains. A more comprehensive understanding of 3- to 5-year-old children's mathematical problem-solving development is likely to result in developing appropriate assessment tools for measuring a variety of 3- to 5-year-olds mathematical skills and in the development of early childhood education instruction to better correspond with preschoolers learning trajectories in mathematics.

Causes of Individual Differences in Mathematical Problem-Solving

Mathematical problem-solving skills may not only vary as a function of age but other factors such as underlying academic and cognitive skills (Lin, 2020). At first, when encountering a mathematical problem, the child needs to understand the problem. As mathematical problems are usually presented either orally (Carpenter et al., 1993; Kyttälä et al., 2014) or in a form of text (Björn et al., 2016), the child is required to comprehend language-based information. For preschoolers, mathematical problems are often presented orally causing *expressive and receptive language skills* to be essential for understanding and stating the answer. Prior research has shown that expressive and receptive language skills are related to closed routine mathematical tasks involving arithmetic operations (Hornburg et al., 2018; Kyttälä et al., 2014).

After understanding the problem, the child needs to build a mental or physical model of the problem situation (Carpenter et al., 1993; Verschaffel, 2000). For example, in the mini-bus example, the child may mentally visualize or manipulate miniature objects of a situation where s/he divides 19 children sitting either by two or three to one seat in a bus. For building and operating with such models, children's *visuospatial abilities* are crucial. A large body of research has shown that visuospatial skills are associated with mathematical skills and predict later problem-solving skills across age groups (Assel et al., 2003; Kyttälä et al., 2014; Raghubar et al., 2010). It has also been argued that especially non-routine mathematical problems require visuospatial skills as tackling these problems may seriously benefit from the ability to visualize the task (Barnes & Raghubar, 2014).

As by definition mathematical problem-solving is applying one's mathematical knowledge, children also need to perform for example calculations or measurements depending on the problem task to arrive at a solution. Naturally, arriving at a correct solution depends on children's ability in the related mathematical skills. Among young children, *early numeracy skills* (i.e., numerical relational and counting skills) have been found as strong indicators of young children's later mathematical knowledge (Aunio & Niemivirta, 2010). Numerical relational skills denote the ability to understand how different numerals, and quantities are related to one another (Purpura & Lonigan, 2013; Wynn, 1998). For example, children's ability to understand the meaning of 'more' or 'less' reflects their numerical relational skills (Aunio & Niemivirta, 2010; Chan et al., 2022). Consequently, numerical relational skills are highly connected to children's ability to combine language with numerical information, which further connects numerical relational skills to mathematical problem-solving skills (Chan et al., 2022).

In addition to numerical relational skills, when solving problems, preschool-aged children also rely heavily on counting-based strategies requiring proficiency in counting skills (Chu et al., 2018). Counting skills require knowing the number word sequence, enumerating objects, and connecting quantities with numerals (Gelman & Gallistel, 1986; Wynn, 1989). In the following example, children may enumerate three times six objects: 'Robin has 3 packages of gum. There are 6 pieces of gum in each package. How many pieces of gum does Robin have altogether?' (Carpenter et al., 1993).

Research among elementary school-aged and older children has implied that the effects underlying cognitive and academic skills may vary as a function of the nature of the problem task (i.e., routine or non-routine; Fitzpatrick et al., 2020; Strohmaier et al., 2021). As prior research investigating the effects of cognitive and academic skills on preschoolers' mathematical problem-solving has involved routine tasks, these relations should be examined in the context of non-routine mathematical problem-solving (Hornburg et al., 2018; Kyttälä et al., 2014). Thus, research concerning the underlying cognitive and academic skills contribution to preschoolers' non-routine mathematical problem-solving is required. Filling this research gap may result in further understanding of the development of 3- to 5-year-old preschool-aged children's mathematical problem-solving skills and related factors, which will provide valuable information for developing early childhood education instruction to better suit the developmental trajectories of preschoolers' mathematical learning.

Present Study

Though some studies have implied that preschool-aged children may be able to solve non-routine mathematical problems (Carpenter et al., 1993; Matallotaki, 2012), no study has investigated children's ability to solve open and closed non-routine problems among 3- to 5-year-olds. In addition to gaining knowledge of preschoolers' ability to solve such problems, understanding the factors related to individual differences is important. Prior research has indicated that expressive and receptive language, visuospatial, and early numeracy skills may contribute to mathematical problem-solving skills among young children in closed routine tasks (Hornburg et al., 2018; Kyttälä et al., 2014). To be able to understand young children's mathematical problem-solving skills profoundly, these associations should also be examined within non-routine problem-solving tasks. Thus, a significant research gap remains in understanding 3- to 5-year-old preschooler's ability to solve non-routine mathematical problems and the sources of individual differences in these skills. The aim of this study was to investigate individual differences in mathematical problem-solving skills among 3- to 5-year-old children through the following research questions:

1. To what extent are 3- to 5-year-old children able to solve open and closed non-routine mathematical problems?
2. To what extent are expressive and receptive language, visuospatial, and early numeracy skills related to 3- to 5-year-old children's mathematical problem-solving skills concurrently and longitudinally?

Materials and Methods

Participants

Participants of this study were drawn from the Active Early Numeracy project (University of Helsinki). A research permit for the project was obtained from one large municipality in Southern Finland, the preschools as well as from the ethical board of the university. The municipality suggested preschools from diverse demographic areas for obtaining a heterogeneous sample. The participating children were then recruited from the preschools that permitted the research

Table 1 Number of children in each age group, mean age, and gender distribution of the sample

	Overall	3-year-olds	4-year-olds	5-year-olds
N	328	115	167	46
Mean age in months (SD)	51.57 (8.02)	42.97 (3.15)	54.22 (3.31)	63.84 (3.12)
Girls	174	63[a]	86[a]	25
Boys	152	51[a]	80[a]	21

[a]Missing information on gender for two participants (one in the 3-year-olds and one in the 4-year-olds age group). *Note.* Age is calculated at the first time point

by sending children's parents a permission form to each child involved in a 3- to 5-year-olds playgroup. Children's parents completed permission forms for the study that included information of the study aims and procedure. Each child whose parent permitted to research was included in the study. Participants were allowed to withdraw from the research at any given time on their own or their parents' initiative.

Participants in this study were 328 3- to 5-year-old preschoolers. The number of children in each age group, mean age, and gender distribution is presented in Table 1. The children were from 16 different preschools representing diverse demographic areas.

Measurement

Expressive and Receptive Language Skills were measured using language skills items from a screening test called 'LENE' (i.e., Leikki-ikäisen neurologinen arvio; Preschool-aged children's neurological evaluation; Valtonen & Mustonen, 2007; Valtonen et al., 2004). LENE tasks included items assessing the understanding of instructions and receptive questions as well as answering receptive questions (Valtonen & Mustonen, 2007). For example, the children were asked to put a toy car in a bag. One point was given for every correct answer. Each age group had different language skills items and a different number of items, and thus, internal consistency is reported for each age group separately. Cronbach's alpha for the expressive and receptive language skills was for 3-year-olds $\alpha = 0.832$, for 4-year-olds $\alpha = 0.841$, and for 5-year-olds $\alpha = 0.876$ in the current sample.

Early numeracy was measured by using the standardized Early Numeracy Test (ENT; Van Luit et al., 2006) which measures children's numerical relational and counting skills. The ENT comprises 40 items, of which 20 measure numerical relational and 20 counting skills. Numerical relational skills items include comparison (comparing between two cardinal, ordinal, or measurement situations), classification (grouping of objects in a class), making correspondence (understanding of one-to-one correspondence with simultaneously presented objects), and seriation (dealing with discrete and ordered entities; Aunio et al., 2006; Van de Rijt et al., 1999). Counting skills items include using number words (using number words in number word sequence up to 20), synchronous and shortened counting (counting object by pointing), resultative counting (accurate counting and last-word response while pointing is not allowed), general knowledge of numbers (application of numeracy understanding; Aunio et al., 2006; Van de Rijt et al., 1999). Some of the items are accompanied by a picture, paper and pencil, or blocks. One point was given for each correct answer. In the present study, the total score of the ENT was used for which the internal consistency was $\alpha = 0.898$.

Visuospatial skills were measured by WISC–III Block design task (Wechsler, 1991). In the block design task, the child is shown either a constructed model or a picture of one- or two-colored blocks arranged to form a pattern. The child is asked to construct the same pattern using four to nine blocks depending on the item, as fast as possible. Each item has a time limit ranging from 30 to 120 s. Depending on the time spent, the items were scored from 0 to 7. The measure has 12 items, but after a mistake (i.e., hitting the time limit or not being able to construct the pattern) in two consecutive tasks, the measurement is suspended. However, in the first three items, the child has two opportunities to construct the pattern and thus, all children conducted at least the first two items before the measurement could be suspended. The internal consistency of the Block design was $\alpha = 0.777$ for the current sample.

Mathematical Problem-Solving was measured by using a measure being developed within the Active Early Numeracy project due to no existing measures for measuring preschoolers' mathematical problem-solving skills being available. The mathematical problem-solving measure was a 21-item interview-based measure (items in Supplementary Information 1 and 2) that was developed to measure non-routine mathematical problem-solving skills. The measure was developed to represent different mathematical domains (i.e., four items in arithmetics, one in pre-algebra, 11 in reasoning, two in combinatorics, and three in geometry). As prior research has found that young children solve mathematical problems through mental, physical, or pictorial modeling (Carpenter et al., 1993),every item in our measure involved a representation of the problem in the form of a picture or material (e.g., coins, blocks) that the child was asked to use in order to solve the problem. All tasks were presented orally, and the child answered either orally or by modeling using the materials. The scoring of the items is presented in Supplementary Information 2. Internal consistency for the mathematical problem-solving measure in time point 1 was $\alpha = 0.796$ and in time point 2 $\alpha = 0.820$.

Procedure

All assessments were conducted individually in a separate room in children's own preschools during regular preschool hours. The assessments were divided into four separate sessions, one for each measure (i.e., expressive and receptive language, visuospatial, early numeracy, and mathematical problem-solving skills) as the participants in our study were so young. Each assessment session lasted a maximum of 30 min. Children's expressive and receptive language, visuospatial, and early numeracy skills were assessed at time point 1 and mathematical problem-solving at both time points. All of the assessments were administered by trained research assistants.

Assessments at time point 1 were conducted from November 2019 to March 2020 and at time point 2 from November 2020 to October 2021.

Data Analysis

Based on their age, children were grouped into 3-, 4- and 5-year-old groups. For the purposes of the second research question, a mean score of expressive and receptive language skills was calculated for all children to be able to evaluate children's language skills using the same metric. This was due to each age group having different (and different number of) items in their measures. For all other variables, sum scores were constructed.

First, descriptive statistics, normality, and reliability (i.e., Cronbach's alpha) were examined for all variables and age groups. In addition, for ensuring that no gender differences appeared, independent samples *t* test was conducted for each age group. To answer the first research question, descriptive statistics for all mathematical problem-solving items were examined for all age groups separately. The second research question was answered in two phases. First, correlations between sum variables (i.e., expressive and receptive language, visuospatial, early numeracy and mathematical problem-solving skills) in time point 1 were examined. Pearson correlation coefficient was used for normally distributed variables and Spearman's rank correlation coefficient for the non-normally distributed variables (± 2; George & Mallery, 2014). Second, a multivariate path analysis with the maximum likelihood estimation with robust standard errors (MLR) in which time point 1 mathematical problem-solving, expressive and receptive language, visuospatial, and early numeracy skills were assigned as independent variables predicting time point 2 mathematical problem-solving skills. Chi-square ($\chi 2$), Comparative Fit Index (CFI), Tucker-Lewis Index (TLI), and the Root Mean Square Error of Approximation (RMSEA) were used as model-fit indices to examine the goodness of model fit. A non-significant chi-square value and values greater than 0.90 for CFI and TLI and smaller than 0.08 for RMSEA are considered as representing adequate model fit (Marsh et al., 2004). All statistical analyses were conducted using IBM SPSS Statistics version 27 and Mplus version 8.6 (Muthén, 1998).

Results

Preliminary Analyses

Descriptive statistics for all variables are presented in Table 2. Internal consistency in the mathematical problem-solving measure from 4-year-olds and 5-year-olds indicated acceptable rates. Due to the low internal consistency of our mathematical problem-solving measure in 3-year-olds' age group, we decided that further statistical analyses from descriptive statistics will be conducted only for 4- and 5-year-olds. Based on the descriptive statistics, mathematical problem-solving skills increase

Table 2 Descriptive statistics and internal consistency for all variables and age groups

	Overall		3-year-olds				4-year-olds			5-year-olds		
	N	M (SD)	α	*n*	M (SD)	α	*n*	M (SD)	α	*n*	M (SD)	α
Time point 1												
Mathematical problem-solving	247	7.53 (4.22)	.797	76	4.29 (2.43)	.535	128	8.02 (3.47)	.695	43	11.84 (4.33)	.772
Early numeracy	293	12.21 (7.16)	.898	97	7.58 (4.03)	.747	152	13.32 (6.93)	.888	44	18.59 (6.70)	.864
Expressive and receptive language skills[a]	292	0.89[a] (0.18[a])		102	0.89[a] (0.15[a])	.832	153	0.88[a] (0.20[a])	.841	37	0.89[a] (0.15[a])	.876
Visuospatial skills	305	6.81 (7.01)	.778	104	2.82 (2.95)	.535	155	7.50 (6.33)	.719	46	13.50 (9.49)	.777
Time point 2												
Mathematical problem-solving[b]	234	13.05 (4.89)	.818	87	10.11 (3.62)	.677	119	13.99 (4.57)	.792	28	18.18 (3.76)	.754

[a]Mean score of language skills

[b]At the second time point, children in all age groups are one year older than at the first time point

with age as 4-year-olds had lower mean than 5-year-olds. Independent samples *t* test revealed that there were no gender differences among 4-year-olds, $t(125) = -0.379$, $p = 0.709$ or 5-year-olds, $t(41) = -0.427$, $p = 0.680$. Missing data analysis by Little's MCAR Test indicated that data was missing completely at random, $p > 0.540$.

Preschoolers' Ability to Solve Open and Closed Non-Routine Mathematical Problems

Descriptive statistics for each mathematical problem-solving item separately can be found in Supplementary Information 1 (item descriptions and scoring in Supplementary Information 2). Children in all age groups were able to solve open (Item 1, 7 and 21) and closed (all other items) non-routine problem-solving tasks. Overall, it seems that 5-year-olds outperformed 4-year-olds in mathematical problem-solving tasks.

Although children were able to solve different mathematical problems, there were items that were very difficult for all or some of the age groups. In line with previous research, items with an accuracy rate below 5% were considered too difficult or an accuracy rate above 95% too easy (Hellstrand, 2020). Across the age groups, a closed mathematical problem involving reasoning, item 18, proved very difficult. In addition, for 3-year-olds, four of the items proved very difficult (Item 9, 16, 17, 18). These problems were all closed and represented reasoning. Four-year-olds had difficulties in items 15, 16, and 18 also representing reasoning and arithmetics, while none of the items proved too difficult for 5-year-olds. In contrast, two items (Item 10 and 19) were too easy for 5-year-olds. Thus, these items that represented geometry and arithmetics proved to be routine tasks for most 5-year-olds.

Cognitive and Academic Predictors of Preschoolers' Mathematical Problem-Solving

For answering the second research question concerning the concurrent associations between mathematical problem-solving, expressive and receptive language, visuospatial, and early numeracy skills in time point 1, correlations were examined. Correlations for 4- and 5-year-olds are presented in Table 3. In time point 1, expressive and receptive language, visuospatial, and early numeracy skills correlated statistically significantly and positively with mathematical problem-solving in 4-year-olds. Within 5-year-olds, only early numeracy and visuospatial skills were statistically significantly and positively correlated with mathematical problem-solving concurrently. From the correlations, the relationship between early numeracy and mathematical problem-solving was the strongest across these age groups.

To examine longitudinal relationships, time point 1 mathematical problem-solving, expressive and receptive language, visuospatial, and early numeracy skills were set as predictors of time point 2 mathematical problem-solving in a multivariate path analysis. As independent samples *t* test indicated that there is a significant difference in mathematical problem-solving skills between 4- and 5-year-olds at time

Table 3 Concurrent and longitudinal correlations between mathematical problem-solving, expressive and receptive language, visuospatial, and early numeracy skills for 4- and 5-year-olds

Overall	1	2	3	4
Mathematical problem-solving skills T1				
Early numeracy	.731**			
Expressive and receptive language skills[a]	.231**[a]	.289**[a]		
Visuospatial skills	.612**	.547**	.232**[a]	
Mathematical problem-solving T2[b]	.711**	.728**	.283**[a]	.553**
4-year-olds				
Mathematical problem-solving skills T1				
Early numeracy	.588**			
Expressive and receptive language skills[a]	.446**[a]	.540**[a]		
Visuospatial skills	.496**	.403**	.401**[a]	
Mathematical problem-solving T2[b]	.609**	.686**	.422**[a]	.386**
5-year-olds				
Mathematical problem-solving skills T1				
Early numeracy	.713**			
Expressive and receptive language skills[a]	.286[a]	.389*[a]		
Visuospatial skills	.454**	.421**	.067[a]	
Mathematical problem-solving T2[b]	.569**	.675**	.296[a]	.075

*$p < .05$

**$p < .001$

[a]Spearman's rank order correlation, otherwise Pearson correlation

[b]At the second time point (T2), all age groups are one year older than at the first time point (T1)

point 1, $t(169) = -5.85$, $p < 0.001$ and at time point 2, $t(145) = -4.50$, $p < 0.001$ we controlled for the children's age. While controlling for the effect of children's age, results of the path model indicated that time point 1 mathematical problem-solving and early numeracy were the only significant predictors of time point 2 mathematical problem-solving skills (Fig. 1). Children's age was found to correlate with other independent variables but not to have a unique contribution to mathematical problem-solving skills in time point 2. As the path model was saturated with no degrees of freedom, the model fit could not be calculated. However, we ran the path model with only the significant paths indicated in Fig. 1 which yielded in good model fit, $\chi^2 = 2.96$ (3), $p = 0.398$, CFI = 1.00, TLI = 1.00, and RMSEA = 0.000, $p = 0.596$. The perfect fit is likely due to a relatively small sample (136 children in time point 2 as 3-year-olds were excluded). The model explained 59% ($R^2 = 0.591$, $p < 0.001$) of variance in time point 2 mathematical problem-solving.

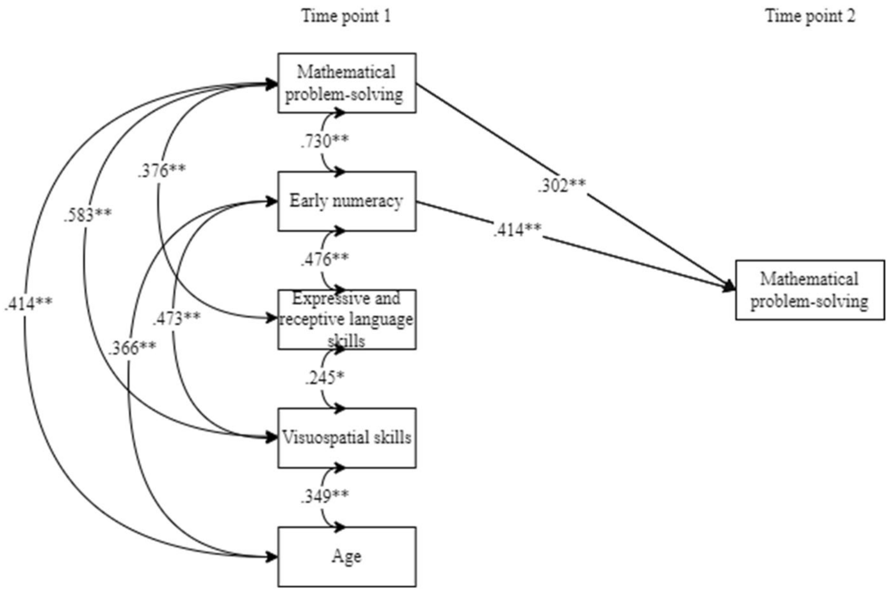

Fig. 1 Significant paths and correlations for predicting time point 2 mathematical problem-solving skills

Discussion

The aim of this study was to investigate individual differences in mathematical problem-solving skills among 3- to 5-year-old children. The results indicated that 4- to 5-year-old preschoolers are able to solve open and closed non-routine mathematical problems that represent a variety of mathematical domains. We also found that 3-year-olds were able to solve a majority of these tasks, though reliability criteria for the measure were not met in this age group. Additionally, though expressive and receptive language, visuospatial, and early numeracy skills were related to mathematical problem-solving skills concurrently, only prior mathematical problem-solving skills and early numeracy predicted mathematical problem-solving longitudinally among 4- to 5-year-olds (3-year-olds were excluded from the analysis due to our measure not meeting reliability criteria). Given the importance of the ability to solve mathematical problems throughout children's lives, our study provides valuable insight into young children's mathematical problem-solving skills. In addition, considering the cumulative nature of mathematical learning, our study provides valuable understanding of factors related to individual differences in mathematical problem-solving that can be potentially used to develop early childhood education instruction.

Prior research has indicated that in addition to routine mathematical tasks, preschoolers may be able to solve closed non-routine problems involving arithmetics (Carpenter et al., 1993; Matalliotaki, 2012), which was supported by our findings. Additionally, our study extends these findings by indicating that 4- to 5-year-olds are able to solve not only closed non-routine problems representing arithmetics but also open non-routine problems representing for example geometry (e.g., forming a shape from tangram pieces or forming symmetrical patterns from shapes) and combinatorics (e.g., forming different combinations of color patterns). Overall, only one task in which children had to fill in missing shapes to form a pattern proved too difficult in these age groups. Additionally, 5-year-olds were shown to be highly successful in one closed task requiring forming symmetrical patterns of shapes and one arithmetic task requiring dividing objects equally. Related to the finding of Matallotaki (2012) that 30–40% of 5- to 6-year-olds were able to solve a similar division task, in our study, almost all 5-year-olds were able to solve such task. This finding highlights that preschool-aged children may already be proficient in mathematical problem-solving tasks involving various mathematical domains such as geometry and division. Despite these promising results, our findings indicated that 3-year-olds mathematical problem-solving could not be reliably measured with the measure used in our study. Future studies should aim to investigate further 3-year-olds mathematical problem-solving skills and how these skills could be measured reliably.

For supporting mathematical problem-solving skills, knowledge of the sources of individual differences such as underlying academic and cognitive skills is vital. Previous investigations have indicated that expressive and receptive language skills, early numeracy, and visuospatial skills are related to routine mathematical problem-solving among preschoolers (Hornburg et al., 2018; Kyttälä et al., 2014). The findings of our study support prior research to the extent that when this relation was examined concurrently, expressive and receptive language, visuospatial, and early numeracy skills were related to non-routine mathematical problem-solving skills among 4-year-olds (Hornburg et al., 2018; Kyttälä et al., 2014). However, among 5-year-olds, only early numeracy and visuospatial skills were related to mathematical problem-solving. Furthermore, our findings extend prior research by indicating that when investigated longitudinally, only prior mathematical problem-solving and early numeracy predicted mathematical problem-solving among 4- and 5-year-olds. This finding indicates that early numeracy skills contribute strongly into the development of mathematical problem-solving skills. Accordingly, numerical relational skills and counting skills that construct our early numeracy measure is closely related skills to mathematical problem-solving. Numerical relational skills are essentially understanding language required in mathematics (e.g., more, less) which is also highly needed when one needs to understand a mathematical problem stated either in a form of text or verbally. In addition, preschoolers often rely on counting-based strategies when solving mathematical problems (Chu et al., 2018), which likely also resembles the relation between early numeracy and mathematical problem-solving skills. As prior research has only addressed preschool-aged children's mathematical problem-solving skills by measuring their knowledge in arithmetic problem tasks (Hornburg et al., 2018; Kyttälä et al., 2014), our findings highlight that the development of mathematical problem-solving skills in various tasks is influenced by children's early numeracy skills. Consequently, future studies aiming to investigate how young children's mathematical problem-solving skills may be supported, should target numerical relational and counting skills in addition to mathematical problem-solving.

Though our study provides novel insight into preschoolers' mathematical problem-solving skills, it has limitations. Expressive and receptive language skills measurement used in the current study was designed as a screener to detect children with language-related difficulties (Valtonen et al., 2004). Thus, many children scored high on this specific measure resulting in a skewed variable. Though this was accounted for by analysis decisions, a greater variability in the measure could have resulted in different findings. Additionally, for practical reasons, our study employed a visuospatial skills measure that was designed for 6- to 16-year-old children, demanding extra caution when interpreting the results. In our sample, however, the internal consistency of the measure was deemed good for 4- and 5-year-olds. Related to sources of individual differences that were not addressed in the current study, previous studies have found that general intelligence as well as executive functions are important aspects of mathematical learning that may well be linked also to non-routine mathematical problem-solving (Hornburg et al., 2018; Kyttälä et al., 2014). Also, as non-symbolic and symbolic number sense have been shown to contribute to young children's mathematical development (Clements & Sarama, 2021; Dehaene, 2011), the relationship between such skills and mathematical problem-solving should also be examined more closely. Consequently, further investigation is needed to determine the extent to which these variables affect non-routine mathematical problem-solving among preschoolers. Our sample of 5-year-olds was also quite small, which limits the possibility of making conclusions about that age group. As our study was not experimental, future studies should examine the possibility of supporting mathematical problem-solving by fostering early numeracy skills with an experimental design.

Taken together, our findings indicate that 4- to 5-year-old children are able to solve a variety of open and closed non-routine mathematical problems involving arithmetics, geometry, pre-algebra, and combinatorics. In addition, our findings showed that 3-year-olds may also be able to solve such problems, though reliability issues with the measure demands caution when interpreting these findings. Only one mathematical problem-solving task, where children had to recognize a missing piece from a pattern of shapes, proved to be too difficult for this age group. As prior to this study, little has been known concerning the development of preschoolers' mathematical problem-solving skills, and our findings provide important knowledge for future studies to construct appropriate mathematical problem-solving measures and to develop early childhood education instruction to better correspond the developmental stages of preschooler's mathematical problem-solving skills. Additionally, our findings indicate that non-routine mathematical problem-solving skills are influenced by mathematical problem-solving skills directly but also by early numeracy skills. Based on our findings, especially from the point of view of educational practitioners, it is important that mathematical problem-solving and early numeracy skills become and remain as a key part of early childhood education.

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Declarations

Conflicts of Interest We have no conflicts of interest to disclose.

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