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# Numerical Modelling of Blast Furnace — Evolution and Recent Trends

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## Abstract

The ironmaking blast furnace has evolved over the years from a small batch processing pig iron unit to a giant hot metal producer. In a journey of centuries, it has gone through many technological changes, which made it a fit-for-purpose till today. However, for the first time in history, its existence is being challenged by other iron making processes, due to its high carbon footprint compared to others. Many researchers are working on adapting the existing blast furnace process to meet the new challenges. Modeling has played a crucial role in new adaptations of the process. The present review has been dedicated to historical developments of blast furnace modeling, which is a reason for the success and competitiveness of the blast furnace process. The models are broadly classified into comprehensive, zone-specific, and data-driven models. The comprehensive models are further classified into lumped models, 1-D, 2-D and 3-D steady state and transient models, and CFD-DEM models. For the purpose of brevity, the zone-specific models are exemplified by burden distribution models only. Finally, developments in data-driven models and their applications are discussed, before some general conclusions are presented.

## 1. Introduction

Human civilization started using iron as early as 1500 BC [1,2,3]. Archeological evidences show that production of molten iron started first in China around 800 BC [4]. Since then the iron production technology has evolved continuously. Interestingly, the idea of counter current gas-solid shaft reactor developed during those times is still valid in modern blast furnaces. One can say that forerunners of the modern blast furnace date to the beginning of 18th century when coke was used for the first time as fuel [5]. Subsequently, at the beginning of 19th century cold blast was replaced by hot blast [6]. Industrial revolution with the advent of the steam engine resulted in large industrial production of iron. Till present the blast furnace has remained as one of the prime producers of liquid iron from iron ore. Though many alternate processes have challenged the existence of the blast furnace, it has been adapted to the needs of the time and has remained thermally, chemically and economically efficient.

In this journey, one of most revolutionary understandings were gained during dissection studies. One of the first experimental studies where nitrogen was blown through the tuyeres in an experimental blast furnace was in the University of Minnesota [7]. From 1970, for a period of 10 years, Japanese researchers quenched four experimental blast furnaces and nine industrial furnaces [8]. One of the most important discoveries from these studies was the existence of cohesive or fusion zone in the furnace. These studies have also provided insight into the importance of burden distribution, furnace stabilization, composition of hot metal, especially low Si hot metal production, and circulation of alkali-containing materials in the furnace. Simultaneously, efforts were on also to develop mathematical models to predict process performance, process control and process understanding.

In the current situation, blast furnace operations face multiple challenges in terms of carbon footprint, lower grades of iron ore, cost of production and development of efficient alternative ironmaking technologies, etc. Evolutions from the early stages to the modern-day blast furnaces have been greatly influenced by physical and mathematical modeling methodologies and it will continue to do the same in near future. It is therefore important to review the past and current trends in modeling of the blast furnace, so a brief review is presented in this paper.

## 2. Classification of Blast Furnace Models

Being a complex reactor with different zones with a multitude of the phenomena occurring simultaneously, the blast furnace has been modelled in different ways. Some investigators have focused on specific parts, or specific phenomena, while others have developed over-all models attempting to describe the whole system at hand. Often, these approaches must be merged. For example, understanding the burden distribution at the top of the furnace, where the raw material is charged, is important in itself for the operator for furnace control. On the other hand, the results of a burden distribution model can be used as boundary conditions for a shaft or an over-all model. Considering the obvious challenges in developing a detailed model of the whole process that would accurately predict its static and dynamic performance, the two approaches must rather be seen as complementary than competing.

For the purpose of the present paper, the review has been divided into three categories with representative examples taken from each:

- **Comprehensive models** 0-D, 1-D, 2-D, 3-D models with steady state or transient conditions predicting the temperature, composition and flow profiles, and CFD-DEM models
- **Zone-Specific Models** – exemplified by burden distribution models
- **Data-Driven Models** – Heat level prediction, stove temperature profiles, anomaly detection, etc.

It should be stressed that the review merely provides a historical overview, focusing on the long-term developments in the field, and it is by no means claimed to be exhaustive. The review is not organized according to the mathematical techniques applied in the modeling, since an excellent review based on this point of view is presented by Kuang et al. [9].

## 3. Comprehensive Models

### a) Heat and mass balance or lumped models

Any fundamental analysis of a reactor starts with overall mass and heat balances that consider the reactor inputs and outputs. The blast furnace is not an exception. In 1927, Reichardt [10] developed a model

along these lines and proposed a graphical representation for aiding the analysis. This model was extensively used in analyzing blast furnace performance till 1970's. Based on the Reichardt diagram, Rist and co-workers (see, e.g., [11]) proposed a more sophisticated model in 1965 with graphic representation popularly known as the Rist diagram as shown in Fig. 1a. This model along with dissection studies paved the way to a systematic description of blast furnace phenomena, i.e., description of different zones such as raceway, dead man, dripping zone, fusion zone, direct reduction zone, thermal reserve zone, chemical reserve zone and the top reduction zone. Subsequently, Peacey and Davenport [12] published a textbook on blast furnace entitled "Theory and practice of the iron blast furnace", a beautifully written book condensing the findings of the earlier analysis and providing explicit equations how to implement the model. Despite being a complex reactor, in this book, authors bring out a very unambiguous, crisp and quantitative picture of blast furnace phenomena using the Rist approach. Since then, blast furnace operators have extensively used the Rist model to analyze the furnace performances, operational issues and to theoretically explore innovative ideas.

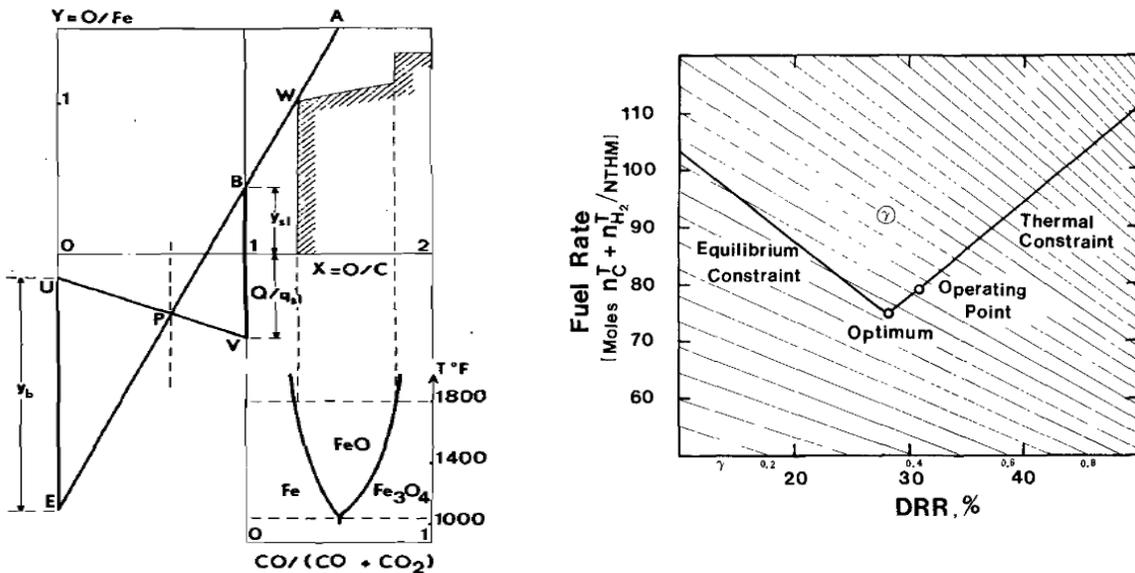


Figure 1: a) Rist diagram showing ideal line, Rist et al. [11], b) C-DRR diagram with operating point, Kundrat [16]

The Rist model works on the premise that there exists a Thermal Reserve Zone (TRZ) of known temperature and Chemical Reserve Zone (CRZ) of known gas composition. The gas composition at the CRZ is determined based on the assumption that wusite is in equilibrium with CO, CO<sub>2</sub>, H<sub>2</sub>, H<sub>2</sub>O and solid Fe. The blast furnace is conceptually divided into two reactors - one below the CRZ and the other above

the CRZ. By performing mass balance (primarily oxygen, carbon and hydrogen balance) and the heat balance, the minimum coke rate and corresponding blast input are computed for given raw material composition, tuyere injections in terms of blast temperature, moisture content, pulverized coal and other hydrocarbon injectants and oxygen enrichment, desired metal composition and desired slag basicity. The computed results are represented graphically in an operating diagram as illustrated in Fig. 1a. In the graphs the variables plotted as  $(O+H_2)/(C+H_2)$  (oxygen by carbon in the gas) vs  $(O+H_2)/Fe$  (Oxygen by Iron in the solids) along the x and y axis, respectively. As shown in the figure, the dotted line shows the line that corresponds to the ideal blast furnace performance given by the Rist model. Additionally, in the graph, the actual operating blast furnace data is superimposed and deviation of this line from the ideal line can be quantified as the efficiency of the furnace.

This is a powerful tool for the operator to assess the performance of blast furnace and can guide the operator towards efficient operation. There are also efforts to predict the hot metal temperature and composition based on the deviation of the actual furnace performance from the ideal performance line. A noteworthy fact is that such ideas were already elaborated in the 1960's, and brought into practical application by IRSID in France, CRM in Belgium and by researchers at Hoogovens in the Netherlands, as reported in classical papers by Staib, Michard, van Langen and co-workers [13,14,15].

Models based on the concept behind the Rist diagram have later been used for a multitude of purposes. Kundrat [16] used a C-DRR diagram (shown in Fig. 1b) to depict the BF performance along with chemical and thermal equilibrium lines and also developed an analytical expression for calculation of coke rate. Pettersson et al. [17] used the concept for cost optimization at the operational level. Furthermore, this model has also been extensively used in exploring new possibilities such as hot reducing gas injection [18], top gas recycle [19,20], natural gas injection both in the bosh and in the stack [21].

#### **b) Comprehensive 1-D Models**

The first attempts to describe the vertical distribution of state variables in the blast furnace, with particular focus on the conditions in the dry part, i.e., shaft, were made in the 1960's. Koump et al. [22] derived a static model of the lumpy zone, considering the overall reduction of hematite to metallic iron by a topo-chemical unreacted core model. Coke gasification was also considered in the model by including the solution-loss reaction. The model was solved from the top downwards, using solid

temperatures and composition as the boundary conditions. The authors adjusted the heat transfer coefficient to get “profiles in agreement with measurements”.

The model proposed by Yagi, Muchi and co-workers [23-25], the first of its kind, is a steady state model. The critical assumptions made in this model were constant packed bed voidage, lumped solids and liquids, etc. The heat, material and force balances for gas and solid particles over a differential height were stated and solved to describe the internal state of the furnace. The model consists of ten first-order ordinary differential equations solved numerically along with three algebraic equations. The model was used to demonstrate the effect of various operating techniques such as high-top pressure, change in particle diameter of iron ore, high-temperature blast, and oxygen enriched blast with the goal to arrive at optimum operating conditions. A counterpart of the model where the horizontal layers of alternate coke and ore were considered explicitly was later proposed by Kuwabara and Muchi [26]. This model has acted as a basis for more sophisticated models described in the forthcoming subsections on 2-D and 3-D models.

The original 1-D model developed by Muchi and co-workers did not treat the behaviour of molten materials, but the model was later extended by Togino et al. [27] to consider the conditions below the melting line. Partly to address these two weaknesses, Fielden and Wood [28] developed a model for predicting the transient behaviour and the hot metal composition as well. The authors discretized the furnace into a number of layers, which were shifted after a time corresponding to the residence time. Since the residence time of the gas differs greatly from that of the burden, a compromise was made. The model was illustrated on a number of cases, including changes after steps in the blast temperature and ore-to-coke ratio, and the results were found to be in general agreement with measured responses. Though the model was capable of predicting the hot metal composition well, the profiles of gas and solid temperature showed a non-physical decrease in temperature in certain regions.

As a result of a compilation of modelling efforts in Germany, von Bogdandy et al. [29] reported a kinetic-dynamic simulation (KDS) model in 1970. Special attention was paid to the formulation of the reaction kinetics based on extensive laboratory tests of iron ore reduction. The model also considered the solution-loss reaction, coke combustion, silica reduction and the water-gas shift reaction. This detailed model was reported in several later publications illustrating simulated dynamic responses after changes in central variables [30,31,32].

By 1973, Flierman and Oderkerk [33] from Hoogovens also presented a dynamic, but quite simplified, model of the blast furnace and used it successfully to survey possibilities of partially replacing coke by injected heavy oil. It was also used to study the dynamic response of the furnace for step changes in operating conditions.

Also, the dynamic model proposed by Hatano et al. [34,35] assumes that the state variables are uniformly distributed in the radial direction of the blast furnace. By contrast to the model by Muchi and Yagi, the reaction kinetics was described by an unreacted core model with two reaction interfaces. The transitional behaviour in a blast furnace was expressed as a set of partial differential equations for each state variable. Then the blast furnace was divided into finite layers, each holding ore and coke. The time step was determined such that the blast rate over the time would supply the necessary quantity of oxygen for consumption of carbon in the unit layer at tuyere level, where the boundary conditions were calculated from input blast conditions. The equations were numerically solved from the tuyere level to the stockline, yielding results for steady state as well as unsteady state operation. For the blow-in operation of the blast furnace, the initial condition of solids in the furnace was given from the actual results of filling of coke and burden before the blow-in. After this the blow-in operation was simulated according to the schedule of the increase in blast rate and blast temperature.

Static 1-D models were also developed by several authors. Saxén [36,37] developed a simulation model particularly for on-line purposes, paying attention to the use of industrial data as boundary conditions. To address the problem of inconsistencies in the measurements, a reconciliation of the measurement data by an interface routine was applied, yielding boundary conditions that obey the conservation laws of mass and energy. The model, which is based on a discretization of the furnace into control volumes, where the chemical reaction rates are expressed by an “approach to equilibrium” method, was adapted to real data by estimation of two key parameters, describing the heat transfer and reaction rates. The model was used to evaluate the current state of the process, but also to predict the effect of operating actions on the performance of the furnace.

Bi et al. [38, 39] extended the model by Yagi et al. to 14 process variables, which include temperature of the gas and condensed phases, composition, volumetric flow rate, density and pressure of the gas phase, degree of iron ore reduction, limestone decomposition and coke solution loss. However, the authors used more sensor information at the top and bottom to get the boundary conditions for the model, e.g., top gas pressure, temperature and composition, operating parameters, hot metal and slag composition

and temperature, and production rates. In order to utilize information from the shaft sensor commonly installed in the lumpy zone, the lumpy zone was divided into two sub-zones above and below the shaft sensor. Another zone, called softening zone, was considered to begin from where the solution loss reaction starts till melting begins. Heat loss from the furnace wall, interior heat transfer phenomena and the porosity of the ore after softening which affects the rate of indirect reduction were considered uncertain factors and chosen to be corrected by three correction factors in order to satisfy the boundary conditions and two additional conditions. The model, which did not consider liquid as a separate phase, could predict the presence of a thermal reserve zone between 950 °C to 1000 °C in the lumpy zone, and the appearance of a chemical reserve zone.

Dobroskok et al. [40] from MISA institute in Moscow developed a model named SCAMPBF, which is known to (have) work(ed) on-line on five Russian blast furnaces. The main difference of this model, compared to most other models, is the fact that it is a diagnosis model and not predictive model: a predictive model can work without a blast furnace by feeding it with the same inputs (burden, blast), but a diagnosis model needs process data, including the outputs (e.g., top gas, hot metal). By contrast, the diagnosis model provides an interpretation of the true state of the process. In this respect, SCAMPBF resembles the model developed by Saxén [37].

Abhale [41] developed 1-D model and faced similar unrealistic temperature behaviour as those obtained by Fielden and Wood [28] shown in Fig. 2a, where the temperatures locally decrease. Also, other simulation models (e.g., [31,37]) occasionally yield similar profiles. The issue was resolved, as shown in Fig. 2b, when the authors incorporated the transfer of heat from solids to gas, which is equivalent of sensible heat of oxygen being removed from the solids and transferred to the gas. Many authors either paid scant attention to this consideration or did not give importance to this term during the formulations of the enthalpy equations of the three phases. Another distinct feature of model by Abhale [41] was that the raceway calculations and the 1-D model were coupled, which provided boundary condition for solids based on the amount of carbon consumed by combustion. This is important when the solids descent rate is unknown (e.g., in scenario analysis) and the production rate is an output of the model.

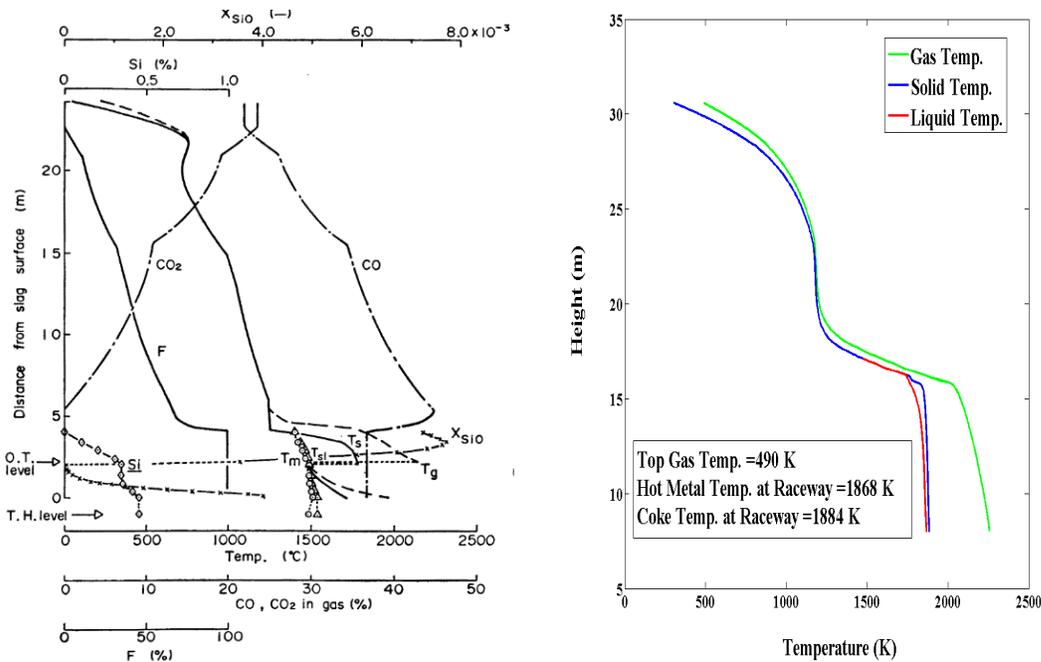


Figure 2: a) Simulation of Chiba No. 3 blast furnace by Fielden and Wood [28],  
 b) Thermal profiles simulated by Abhale [41].

A number of 1-D models have also been developed to address the problem of controlling the furnace by describing the dynamics in a sufficiently detailed, but yet comprehensive, way to allow for on-line use. A few such models will be reviewed here. Hatano et al. [42] developed an approach for describing the dynamic state of the blast furnace under less extreme conditions than those in the blow-in or blow-out analysis described above. This one-dimensional model discretized the furnace volume into five zones, considering preheating, initial ore reduction, indirect reduction and solution loss, direct reduction, as well as combustion of coke and auxiliary reductants. Each zone was represented by average conditions, and the dynamic behaviour is obtained by an updating of these conditions, yielding a matrix formulation of the differential equations. The model was applied to simulate the step responses after changes in the oil rate and was later used to control the oil injection rate [43]. It is quite interesting to note that despite the very appealing properties of the above model, featuring “simple is beautiful”, very few similar attempts have been reported in the literature. This is likely due to the challenge of being able to condense the salient features of the process with enough details but avoiding excessive need of parameters that must be tuned. A recent attempt in this direction was presented by Hashimoto et al. [44], who developed a one-dimensional model dynamic model and applied it to predict the thermal state of the process to be

able to reduce the variation in the hot metal temperature. This is one of the few papers that have successfully applied model-based control of the BF using a first-principles model.

### c) Comprehensive 2-D Models

The first two-dimensional simulation models of the blast furnace were developed in the late 1960's, but the computational power of the computers at that time clearly limited the development. Already in 1969, Lahiri and Seshadri [45] presented a 2-D model of the dry region of the blast furnace, extending to the 1700 K isotherm. The gas was taken to consist of nitrogen, carbon monoxide and carbon dioxide only, and reduction from hematite to wustite, and wustite to metallic iron was considered, also including the solution-loss reaction (so the direct reduction of wustite could also be modelled). The pressure drop was described by the Ergun equation [46]. Imposing symmetry at the furnace center ( $r = 0$ ) and a heat flux at the wall, the arising partial differential equations model were solved by a marching technique. However, the results of the model must be considered rather theoretical since the boundary conditions imposed were artificial and did not reflect the true conditions in the furnace. To the best knowledge of the authors of the present review, the model was not further developed.

To address the problem of the fictitious conditions imposed in the model by Lahiri and Seshadri [45], Muchi and Kuwabara [47] extended their earlier layered 1-D model [26] to two dimensions by considering the (possible) inclination of the burden layers observed in quenched furnaces. The burden layers were assumed to have V-shaped interfaces, with different angles for ore and coke, and the model was again discretized using a consecutive ore and coke layer in each "block". The resulting picture of the internal conditions was found to be in general agreement with findings from the dissections of furnaces, showing a much slower temperature rise at the wall than in the centre. This model thus also provided a view of the cohesive zone in the furnace.

In the 1980's, the first comprehensive 2-D model of the blast furnace was developed by Hatano and Kurita [48]. This steady-state model considers the effect of layer structure on gas flow by using the concept of series and parallel resistances. The solids were assumed to follow predefined streamlines, considering and assumed dead-man shape. Liquid was assumed to flow vertically from where it is formed. A two-interface topo-chemical model was used to describe ore reduction by CO and H<sub>2</sub>. Direct reduction of molten wustite by solid coke, decomposition of limestone, solution-loss reaction, reaction between coke and steam and water gas shift reaction were other reactions considered in the model. Partial

differential equations derived from mass, energy and momentum balances were converted to ordinary differential equations by the method of characteristics. This was the first model which corrected gas-solid heat transfer coefficient significantly above the softening temperature to match the measured top gas and hot metal temperature. Later, most researchers applied similar assumptions, without attempting to clarify the real reason behind this obvious discrepancy of the Ranz-Marshall equation for estimating the heat transfer coefficient [49]. A likely reason is that the effective area of the bed is clearly lower than the apparent value. The validity of the model was confirmed with above burden probe measurements and cohesive zone shape similarity with the quenched blast furnace investigation findings. A major achievement of the simulations was finding a strong correlation between the furnace performance and the radial distribution of the ore-to-coke ratio of the burden at stock-line.

The widely known 'four-fluid model' was initiated by Yagi and his co-workers in the early 1980's at Tohoku University, Japan with the support of ISIJ [50]. Simulations by the model revealed that the layered burden and cohesive zone mainly control the gas flow [51]. This model was further developed by Nogami and co-workers. Descriptions of more detailed solid, gas, liquid and fines cold flow models and interactions among the four phases (gas, solid, liquid, and dust) were modelled and validated by Chen in the early 1990's [52]. A viscous flow model was proposed to describe the solids flow in the blast furnace by introducing the concept of solid viscosity. Austin [53,54] incorporated discontinuous phases in the model which required strong numerical implementation to be able to describe the complex heat and mass transfers between all four phases. Single interface three-stage ore reduction reactions were incorporated along with 14 other reactions. Local reaction rates were calculated to be proportional to the local O/C ratio, which was extrapolated from the top, discretizing the system with 90 grid points in the axial direction and 25 in the radial. Even though the initial development applied the FEM technique, a FVM-based technique was later applied to solve the PDEs. The simulation results showed how different shapes of the cohesive zone can be produced by altering the radial O/C distribution at the top. When the model was extended to include silicon transfer reactions, it was found that compared to the base calculations, the predicted CZ position is higher, and the bosh is cooler when silicon transfer reaction is included. The four fluid model was later extended to analyse transient behaviour by de Castro et al. [55] for high PC injection and NG injections, effect of charging ferro-coke and iron-carbon composites, etc.

Another notable model from Japan is the BRIGHT model developed by Sugiyama et al. [56,57]. The model was used along with the burden distribution model RABIT and raceway model. A three-interface shrinking

core ore reduction reaction model was used, which was later changed to the three-stage single interface reaction model by Naito et al. [58] in the N-BRIGHT version of the model. The vectorial form of the Ergun equation is used to describe the gas pressure loss in the bed. Silicon transfer, reduction degradation and powder accumulation and liquid hold-up models were also incorporated in this version. The computation time required on FACOM M340S computer was reported to be about 65 minutes.

Development of 2-D 'SHAFT' model at BlueScope Steel (formerly BHP) was initiated by Burgess [59] and co-workers at the beginning of the 1980's. The model that was first used in off-line mode till 1996 and thereafter on-line, has been continuously updated with new features, e.g., by Zulli and co-workers [60] and academic researchers mainly from the University of New South Wales. For example, a two-liquid (metal and slag) flow sub-model and the multiple interactions with other phases like liquid hold-up and flooding were introduced in later versions. It is reported that the model has been used as a tool to control heat loads on the lower shaft by acting on the burden distribution. It was demonstrated that the highest production can be achieved when the gas flow was bifurcated around the CZ at the mid-radial location (L or W shaped cohesive zone). This was also found to correspond to a period of strong furnace stability, lowest hot metal silicon content and hot metal temperatures. Dong et al. [61] modified the SHAFT model to obtain a layered treatment of the CZ. Simulations compared different formulations of the cohesive zone, i.e., layered, isotropic, and anisotropic non-layered and their influence on the results. It was demonstrated that the predicted fluid flow and other phenomena within and below the CZ are different for different formulations. Quite naturally, the layered CZ treatment could better predict the role of the zone as a gas distributor and liquid generator than the other methods.

Development of a 2-D steady state BF model for gas distribution and ore reduction, MOGADOR, was jointly initiated in 1995 by (former) Arcelor and Corus in Europe in collaboration with CRM, Belgium. The model, primarily developed by Danloy and co-workers [62,63] predicts the CZ position considering the layered burden distribution. The effect of the layer structure on gas flow was captured by an anisotropic packed bed resistance that was used in the 2-D Ergun equation. The carbon, silicon and manganese transfers to hot metal were neglected in the first approach. A finite difference scheme was applied to discretise the PDEs, using a rectangular 20 x 120 mesh. Though the model was implemented in various European blast furnaces for process diagnosis and scenario analysis like top gas recycling, the model was not found to be grid independent and therefore requires finer mesh for better accuracy. Moreover, the model needs frequent tuning of internal parameters to match the measured outputs.

Another example of 2-D steady state model development is given by the work by Yang et al. [45] at POSCO, South Korea. The finite volume method was employed for solving the mass and energy balance equations. To simplify the calculation procedure, a layer structure is taken to exist in the entire blast furnace based on the top-layer profiles, aligning the grids with the layer profiles and using 25 grid points in the radial direction. This enabled straightforward calculations of the resistances of an isotropic packed bed that was needed to solve the Ergun equation. The model, which considers only two phases, i.e., solids and gas, has been validated by comparing the calculated and observed top gas temperature distribution, yielding good agreement.

Following the developments in India of the early 2-D model by Lahiri et al. [45], IIT Bombay has been working for the past twenty years on modelling of various phenomena in the blast furnace. The notable development happened when the scheme for efficient simulation of gas flow in layered packed bed was developed by Abhale et al. [41, 128]. This gas flow model was then integrated with the main 2-D model. The original codes were not parallelized and there was also room for improvements in the numerical methods used. These drawbacks were later eliminated in the development of 'BlaSim', a 2-D model based on OpenFOAM [65], developed by Tata Steel. The OpenFOAM framework provided efficient solvers and discretization schemes for the solution of the arising PDEs. The model can also be easily extended to 3-D and transient simulations.

Recently, Yu and Shen [66] have presented a 2-D model of the blast furnace considering unmixed ore and coke layers with distinct reactions in the ore and coke layers. A fine-structured grid was used to discretize the domain. After initial rough convergence, all sub-models are rerun to predict the cohesive zone till it stabilizes. As opposed to many other models, this model was able to capture the thermal and chemical reserve zones in a realistic way. The temperature difference between the gas and solid phases was found to be maximum at the top, in the cohesive zone and near the raceway, as expected. The fluctuating iso-lines of gas composition, gas and solid temperature were also well captured by the model. Kuang et al. [67] used a similar approach and integrated a raceway model and shaft model and studied effect of hot charge on blast furnace performance.

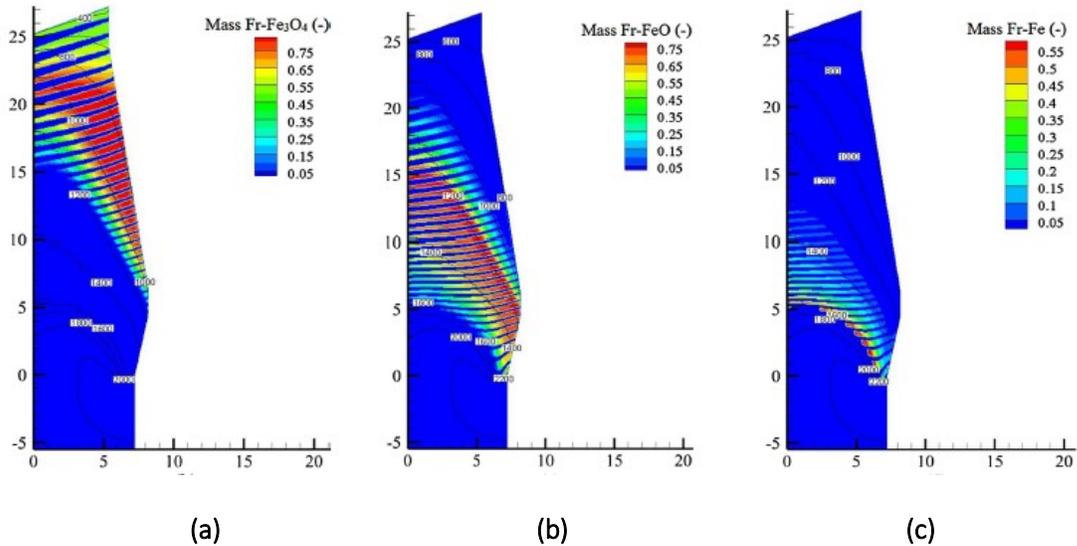


Figure 3 a) Mass fraction of magnetite, b) wustite and c) iron in the model by Yu and Shen [66], considering different reactions in iron- and carbon-bearing layers.

By contrast to the above models, which primarily simulate the conditions independent of measurements, a 2-D shaft model based on measurements in an operating blast furnace is finally mentioned. This model, developed by Kilpinen in the late 1980's [68], can be used for on-line estimation of the thermal and chemical state of the shaft and the position and shape of the CZ. The lumpy zone of the furnace was discretized into several coaxial cylinders, with measurements of gas temperature (and/or composition) by above-burden and in-burden probes. Assuming a stepwise reduction of iron oxides by CO and H<sub>2</sub>, the reaction kinetics was simulated with a three-interface shrinking core model which was developed by a set of laboratory experiments of gaseous reduction designed by a factorial plan to maximize the information from the experiments. The top gas measurements provided upper boundary conditions while the in-burden probe measurements were used for adapting model parameters by a Kalman filter. A set of differential equations were solved numerically for each coaxial part and the vertical coordinate where the temperature of the burden reaches the melting temperature of the reduced iron oxides was interpreted as the upper boundary of the cohesive zone. An assumption of steady state for the solid phase was used at the initiation of the estimator. The model was demonstrated to be able to follow the dynamic operation of the shaft, making it suitable for on-line use as a decision-support tool.

#### d) Comprehensive 3-D Models

The need of 3-D models of the BF is justified by phenomena like tuyere blanking, scaffolding, etc., where axial symmetry cannot be imposed. Initially only 3-D gas flow simulations were carried out, e.g., in the model of Ohno et al. [69]. These studies showed the effect of O/C ratio distribution on the gas flow. Takatani et al. [70] developed the first, comprehensive 3-D transient model which made it possible to make a direct comparison with a real blast furnace in operation. A kinematic model was used to simulate the solids flow in the furnace, while liquid velocities were assumed to be constant above the hearth. Raceway combustion was not included and a perfect mixing of metal in the hearth was assumed, considering heat-loss boundary conditions to be able to predict the hot metal temperature. The authors presented a comparison of the models results with data measured at four different Japanese blast furnaces, showing how the blast furnace geometry can be optimized. Also, dynamic behavior, e.g., at blow-in and blow-off operations, were simulated.

Spalding and Zhubrin [71] developed a four-phase 3-D model named SAFIR using the commercial CFD software PHONEICS. The four phases were 1) gas, 2) fast-moving particles of coal, coke, and liquid droplets suspended in the gas, 3) slower-moving solids consisting of iron ore and coke, and 4) faster-moving liquid metal. The model was applied to describe raceway and dead-man conditions, and the results showed general agreement with present knowledge and understanding in the field. However, the authors recommended an improvement of the solid flow model in the future.

de Castro et al. [72] extended the 2-D model by Yagi et al. [50] to three dimensions, wherein five phases, namely, gas, lump solids, pig iron, molten slag and pulverized coal were considered. To improve the convergence of the model, covariant velocity projections were used in the discretized momentum equations, which gave the best coupling between the velocity and pressure fields. Gas and solid phases were treated as continuous phases, whereas hot metal, slag and pulverized coal were treated as discontinuous phases. In order to calculate the phase volume fractions of the discontinuous phases, continuity equations were used while momentum conservation was used to calculate the velocities. Four different number of raceway configurations were studied, showing that a larger number of raceways leads to more uniform gas flow in the lower region of the furnace.

At Purdue University in the USA, Zhou et al. [73,74] developed a CFD based 3-D shaft simulator along with burden distribution and raceway model. The full Navier-Stokes equation with a turbulence model

was solved for the gas flow. This model is different from many other models with respect to the computation domain, as the shaft simulation and raceway model were assumed to be connected by a horizontal cross section at belly region. This might be considered as one of the limitations of the model as it restricts the cohesive zone root to go below the belly region. A notable feature was that the cohesive zone was defined based on the calculated softening and melting temperatures of local burden composition.

Shen et al. [75] developed a 3-D model using the commercial software package ANSYS-CFX to describe the complex behaviour of solid–gas–liquid in a blast furnace. A viscous flow model was used for the solids flow, whereas the liquid flow was based on a force-balance equation. The increase in resistance in ore layers of cohesive zone was captured by an anisotropic treatment. The model makes use of about 50,000 mesh points in the simulations. The authors found that a 2-D simulation of the liquid fraction distribution in the vicinity of the raceway yields erroneous results and that only a 3-D model can capture it correctly, as shown in Fig. 4. In another study [76], results of slot model vs. sector model were compared for given gas and solid input rates and it was found that the sector model provides slightly better results than the slot model.

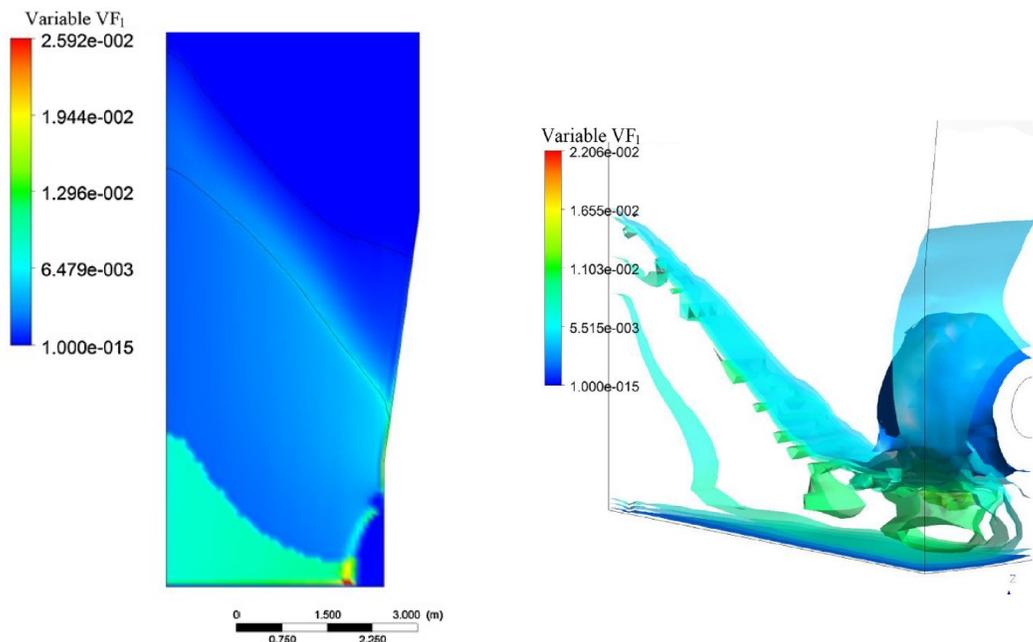


Figure 4: Volume fraction of liquid in a) 2D slot model and b) 3D model by Shen et al. [75].

### e) Comprehensive CFD-DEM Models

Recently, the increased computation speed has allowed for simulations with the solids phase described by discrete elements, e.g., by the Discrete Element Method (DEM) [77], combined with computational fluid dynamics. However, because the number of true particles in the blast furnace exceed a billion, either down-scaled furnace geometry or up-scaled particle size is usually applied. The DEM approach was first applied in the blast furnace to separate sub-problem, such as burden distribution (see Section 4), dead-man formation or raceway modeling, but lately some approaches to describe the whole furnace have been reported. In what follows, two models which illustrate the present state-of-the-art in the field are described.

Bambauer et al. [78] presented an extensive analysis of a three-dimensional CFD-DEM model of a blast furnace in down-scaled geometry. The model considers the flow of solids and gas but disregards thermal effects, i.e., the model is isothermal. Special attention was focused on selecting the simulation conditions appropriately (e.g., the gas supply rate) to reflect the true conditions as well as possible, e.g., by guaranteeing that key dimensionless quantities have similar values in the real and simulated processes. The cohesive zone is fixed and modeled by changing the permeability of the descending layers, converting the ore particles into one liquid phase at the lower end of the CZ. The coke particles are slowly removed from the raceway zones, which have a fixed size. The investigators compared 2-D (i.e., slot) and 3-D formulations of the problem, and furthermore also focused particular attention on the conditions in the hearth, where the effect of liquid level on the floating of the dead man was examined. As the model is dynamic, an attempt was finally made to describe the evolution of the liquid level during tapping, considering the (possible) motion of the dead man.

As another example of the present state-of-the-art in the field, a model termed “virtual blast furnace” developed by Yu and co-workers at Monash University, Australia, is described. The paper by Hou et al. [79] reports an application of the model to simulate the conditions in the experimental blast furnace (EBF) in Luleå, Sweden, in two dimensions, considering non-isothermal conditions and the main chemical reactions, running the model to a quasi-steady state. This sophisticated model can describe the over-all conditions in the furnace, verified by comparison of the simulated profiles with probe measurements in the EBF. By contrast to the model by Bambauer et al. [78], the present model can predict the level and shape of the cohesive zone. On the other hand, the simulated conditions in the hearth are simpler.

In summary, these two examples clearly illustrate that models based on the CFD-DEM approach are gradually approaching the levels of continuum-based models. Even though the computation time will remain a limiting factor for models in the former category, the advantages offered by their better ability to simulate the true phenomena and behavior in certain regions (e.g., CZ, raceway and hearth) will most likely in the long run favor the development of models of the latter category.

#### **4. Zone-Specific Models**

In order to limit the extent of this review, only one example on zone-specific models is presented, i.e., models focused on burden distribution in the blast furnace. It should be noted that an extensive literature exists on models of other regions, e.g., the cohesive zone, raceways and the hearth. Ghosh et al. [79] present a detailed review of dripping zone modeling techniques, raceway model is surveyed in the report by Zhou [80] while other local models, including models with CFD-DEM coupling, have been reviewed by Ueda et al. [81].

##### **Burden distribution models**

The supply of the raw materials, primarily (preprocessed) ore and coke, at the furnace top — charging — has proven to be of central importance for controlling and optimizing the state of process. This is so because the ore-to-coke ratio affects the production and fuel rate of the process, and the radial distribution of the burden materials strongly influence the conditions in the upper part of the furnace, where the indirect reduction of iron ore occurs. Because of the importance of controlling the distribution, modern blast furnaces are equipped with bell-less top (BLT) charging systems, while the use of the older and traditional bell-top charging is limited to old furnaces in use. Therefore, this section gives emphasis on simulations of BLT systems.

Burden distribution models can be classified into classical models and DEM-based models. For both kinds of models, the following aspects are important:

- Flow in hopper and down-comer
- Flow over the chute surface

- Trajectory of stream of particles falling from chute
- Formation of burden ring(s) on the burden surface
- Modification in base and top surface due to push, percolation, etc.
- Evolution of the layers during burden descent

Due to the complexity of the particulate flow, a primary way of investigating aspects related to charging has been small-scale experiments, in cylindrical or semi-cylindrical pilot models, where the arising burden layers can be studied. However, burden distribution experiments are laborious and it is difficult to analyze the internal conditions of the arising bed. Therefore, mathematical modeling is a viable option to gain understanding of the conditions under and after charging of rings of burden. The first detailed classical model of burden distribution was RABIT (RADial Burden distribution Index Theoretical) model developed by Nippon Steel in the 1980's. The model is useful in itself, but also for giving important boundary and initial conditions to comprehensive 2-D and 3-D models of the blast furnace. Initially, the model was based on a description of static base layers in bell-top charging, but it was later modified by Kajiwara et al. [82]. The enhanced model [83, 84] considered the effects of 1) mixed layer formation during ore dumping on a coke layer, 2) decrease in deposit angle due to gas flow, and 3) variation in burden descent velocity in the radial direction. Among these three factors, the mixed layer formation was found to have the most significant effect on the burden distribution.

Hattori et al. [85] developed a burden distribution model in the early 1990's for large blast furnaces with BLT. A physical model of 1:10 scale was also built, where the falling trajectories and layer profiles were measured. Findings from the experiments were used to tune the model equations for falling trajectories and layer profiles formed. The model was implemented at Keihin BF 2 of NKK, and its application was reported to have contributed to improved productivity and stability of the operation.

Radhakrishnan and Maruthy Ram [86] performed measurements of falling particle trajectories in BF No. 5 equipped with BLT charging system at Bhilai Steel Plant and subsequently developed a mathematical model to simulate the formation of layer profiles. The formation of the ring-shaped hill of material was modeled considering the angle of repose, rolling behavior of particles on an inclined surface and layer penetration. The model was used in an optimization routine to determine the charging sequence to be followed to obtain a desired stock-line profile.

Saxén et al. [87] developed a burden distribution model based on radar measurement of stock-line at one point in combination with dump volume calculation and repose angle of the material for a bell-type charging system. A simple scheme was adopted to allow for a tracking of the descending layers in the shaft under the assumption that the layer volumes stay constant. The model was later extended to BLT charging and by a more sophisticated procedure for tracking the descending layers [88]. This model was also used together with a simple shaft model to evolve charging programs [89] (see Section 4). A verification of the model's results was provided by Li et al. [90].

In order to develop a burden distribution model for compact bell less top charging system at Tata Steel, Nag and Koranne [91] performed falling particle trajectory trials at the F blast furnace and modeled these based on a single particle force balance, tuning the model to the measurements. Based on 1:10 scale experiments with different burden materials, Nag et al. [92] later proposed a methodology for estimation of top layer profiles for given inclination of the base layer and developed a top layer profile prediction model. Its results were validated using profile meter data from the real furnace. The model has been incorporated in the Level-2 system of all blast furnaces in Jamshedpur, India for operator guidance. The trajectory model is tuned whenever trajectory measurements are conducted during shutdown.

Shi et al. [93] proposed new equations for calculation of the inner and outer repose angles based on the burden descent rate at the top. A stock line profile prediction model was modified to incorporate the changes in repose angle equations, as well as calculation of falling point by descending the layers (formed after one charging sequence completed) before new materials are charged. It was concluded that the influence of burden descent is significant. The descent procedure of the layers developed by Fu et al. [91] is similar to the method proposed by Mitra and Saxén [88]. The model was validated by experimental data.

Park et al. [95] developed the similar model for Hyundai Steel with equations partly based on scaled model experiments and DEM simulations. The layer descent model bears strong resemblance with models proposed by other authors [88,94], but, by contrast to the models reviewed above, this model also estimates the variation of particle diameters in radial direction based on their volume fraction variation.

Even though the classical models have been quite successful in predicting the layer profiles accurately, these cannot usually accurately predict particle size segregation and void distributions. Furthermore, effects such as coke push and local collapse of layers cannot be captured by traditional first-principle models. Due to the importance of size segregation, layer redistribution and voidage distribution on the arising gas flow, and therefore also on the shaft performance, modeling that considers the behavior of discrete particles, e.g., DEM, has been studied extensively in the past two decades despite their large computational requirements.

The first DEM-based studies of burden distribution problems were published by Mio et al. [96-98], considering the flow of particles from the bunker onto the chute, along it and the trajectory of the materials to eventually distribute on the burden surface. These first efforts have been followed by many papers focusing on different aspects of burden charging and distribution in the blast furnace. These included burden flow during discharge of bunkers, flow on and from the rotating chute, burden layer formation, and burden descent [99-104]. Phenomena that are particularly difficult to study by traditional models but where DEM-based approaches have met success include percolation of smaller particles (e.g., pellets) in layers formed by larger ones (e.g., coke) [105] and the effect of particle shape on percolation [106], gouge formation at impact of the falling stream [107] and coke-push or coke-collapse effects [108,109,110]. As an example of the above, Mitra and Saxén [109,111] simulated the burden distribution in a small-scale charging rig with rotating chute arrangement by DEM and made comparison with experimental results. Pellets were approximated as spherical particles and coke by clumped-sphere particles. The push effect was evident when a ring of pellets was charged on top of the coke surface, as seen in Fig. 5.

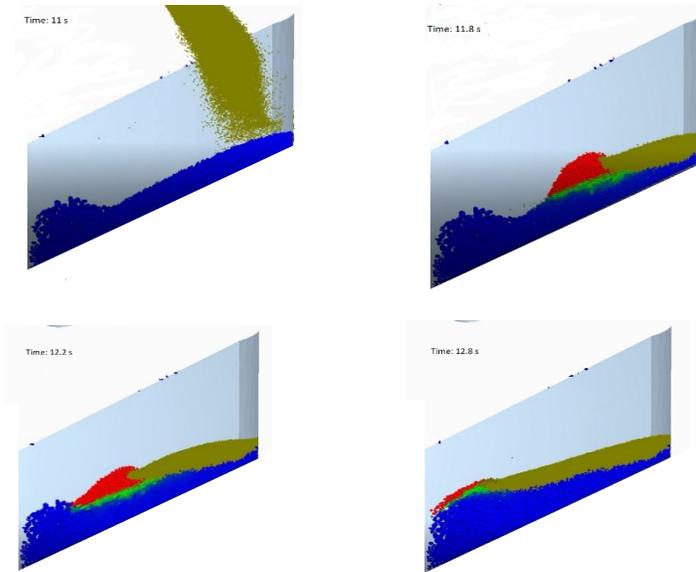


Figure 5: Pellets pushing coke particles (red colored) towards center (images in sequence from left to right and top to bottom) in DEM [111]

## 5. Data-Driven Models

Data driven models correspond to black-box models based solely on operation data to estimate or predict an important quantity, often named Key Performance Index (KPI). In the BF process, KPIs are, e.g., hot metal temperature, Si content, fuel rate, production rate, and gas utilization. Data-driven models use various data analytics tools like artificial neural networks (ANN), principal component analysis (PCA), partial least squares (PL), support vector machines (SVM) or data classification techniques for the purpose. A joint name often used today for such models is Machine Learning (ML), even though the original definition of ML was narrow. Combinations of such models with fuzzy logic have been used to develop expert systems for predictive control, whereas advanced non-linear optimization routines, such as Genetic Algorithms (GA) are being used for complex multivariable optimization tasks, e.g., for minimizing the fuel rate and simultaneously maximizing the production rate. Some of these techniques are also being used for anomaly predictions.

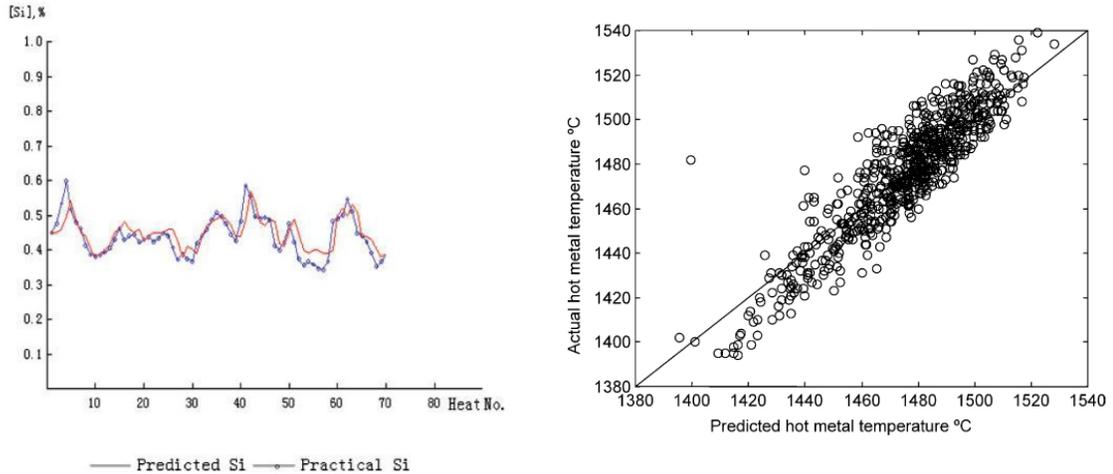
Autoregressive models for prediction of the silicon content of hot metal have been developed since the 1970's [112-114] and the first attempts with neural networks were made in the 1990's [115,116]. Initial attempts of silicon prediction by neural networks in the early 1990's met with some success [115] essentially implementing a nonlinear autoregressive vector model with exogenous inputs (NARX). The

predictions were found to improve by using thermal indices and lead indicators in the model. This approach was later followed up by similar black-box modeling activities [116]. Attempts of applying neural networks to analyze BF operations were also made at MIT, USA, in 1995 in collaboration with Tata Steel India, where Angela [117] used Stuttgart Neural Network Simulator (SNNS) to predict the HMT and Si content in hot metal. The issue of irregular measurement of HMT and Si was resolved by linearization of data and moving average techniques. Despite difficulties in prediction, a reasonable accuracy was achieved by using the predicted variable from the previous step along with 11 other input variables.

Radhakrishnan and Mohammed [118] utilized a neural network based predictive system to work as a soft sensor to provide a real time estimate of the silicon, as well as sulfur content in the hot metal of the blast furnace. The authors suggest dividing the inputs into two categories, one with the “top” variables and the other with the “blast” variables. The variables in the first category show time lags of 6-8 hours whereas those in the second category show more immediate effect on the furnace. After the complete design of the network a sensitivity analysis was done to remove less significant inputs, yielding improved prediction accuracy.

Chen [119] developed predictive system for blast furnaces by integrating a neural network with qualitative analysis. The predictive system predicts the silicon content in hot metal which is used as an index to represent the thermal state of a blast furnace. The qualitative representation is achieved through causal analysis in which the systems with incomplete quantitative information is presented in the form of qualitative models. The actual and predicted silicon content are illustrated in Fig. 6a.

Jimenez et al. [120] designed a system for predicting the temporal evolution of the hot metal temperature by using a NARX and NOE (Non-linear Output Error) models. In the NARX model the forecasted value of variable is considered as a function of the values taken by this variable at the previous sampling times and of the values of the input variables, while in the NOE model the last output is substituted by the model’s own previous output. The authors also used time as an explicit variable to overcome the irregularity caused by irregular taps. The authors suggested to retrain the model every time new output is available to keep the model up-to-date. A comparison of the predicted and actual hot metal temperature is illustrated in Fig. 6b. As a conclusion of the study, the NARX model predictions were found to be more accurate than the NOE predictions.



**Figure 6: Data-driven model for a) Si prediction by Chen et al. [119], b) Hot metal temperature prediction by Jimenez et al. [120].**

A parsimonious model of the silicon content was developed by Saxén and Pettersson [121] by applying a neural network pruning method to eliminate less significant inputs among a large number of candidates, including different time lags for each. It was demonstrated that “blast” variables, and in particular, the heat losses in the tuyeres were significant indicators with time lags of a few hours for predicting the silicon content.

Wang et al. [122] utilized the concept of Mutual Information (MI) in designing a system to predict the silicon content. MI measures the general dependence of random variables without making any assumptions about the nature of their underlying relationships. Initially, a set of 15 input variables accompanied with the rate of change of these variables with respect to time were considered as the inputs for the predictive system, then the least significant variables were removed using MI. Finally, a set of nine variables were found sufficient for designing a predictive system using a support vector machine model. When compared to a standard back-propagation network, the prediction based on SVM in combination with MI was found to be more accurate.

In summary, a large number of data-driven models for hot metal silicon (or temperature) prediction have been presented. A quite recent review [123] summarizes the modeling efforts in the field up to 2013, but during the last five years many more models have been presented, particularly by Chinese authors.

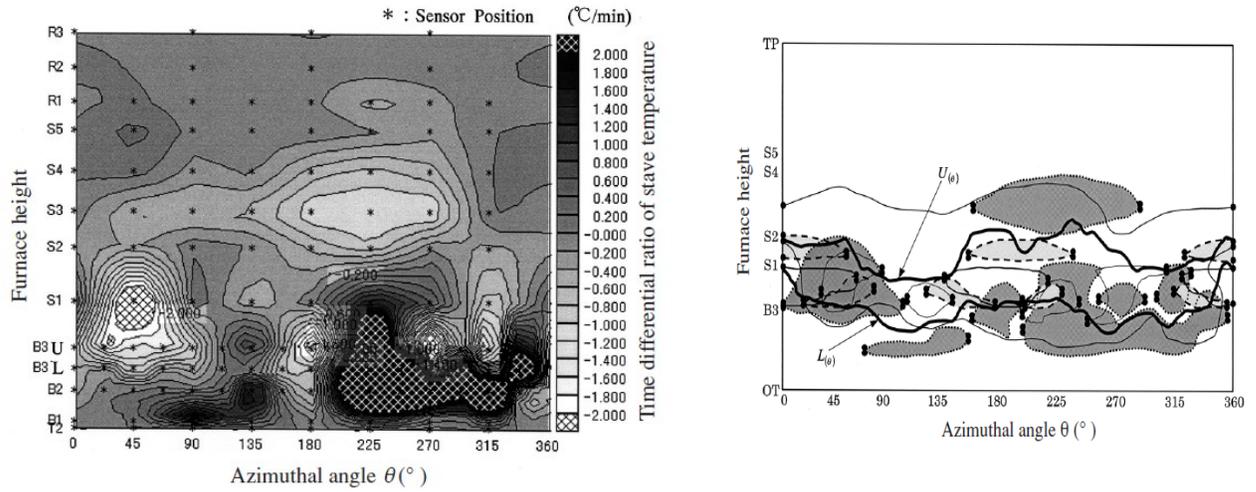
Also, other types of problems in the blast furnace have been tackled by data-driven approaches. Pettersson et al. [124] used genetic algorithms as a tool to evolve neural networks to predict longer-term (week-wise) changes in the hot metal C, Si and S based on a set of potential inputs measured at the BF. A predator-prey algorithm was applied to optimize the individuals in the population, using a bi-objective formulation where both the prediction error and the network size (expressed as the number of weights) were minimized simultaneously. Since each prey has an individual weighing of the two objectives, the procedure evolves candidates that can be used to create Pareto fronts in the space of the two objectives. The authors stress the importance of formulating the cross-over operator in the GA in a meaningful way in order to guarantee that well-working sub-parts of the networks can be preserved and combined in the evolutionary process.

Otsuka et al. [125] developed a predictive system for forecasting decreasing heat levels in the furnace, which were accompanied with a sudden rise and then a fall of wall temperatures. Unsupervised learning for clustering different heat-drop patterns was applied, followed by a classification of clusters into significant and insignificant heat drops. After classification, a neural network was trained to classify new wall temperature patterns into the derived classes. After training, the network was able to predict emerging temperature drops prior to their occurrence.

Mitra et al. [89] used evolutionary and multi-objective genetic algorithms to evolve charging matrices that yield a desired gas distribution at the furnace top. A burden distribution model [88,90] coupled with simplified gas flow distribution was used to generate the large set of data used to train neural networks using the evolutionary approach. The model optimized the charging matrix to match a target radial distribution of the top gas temperature, subject to constraints on the overall pressure-drop and ore-to-coke ratio.

Ito et al. [126] used wall pressure measurements and stave temperature data to predict the location of the root of the cohesive zone along the circumference of the blast furnace. Threshold values of the time differentials of stave temperatures and for the spatial differential vector of shaft pressure were used for the detection, applying a forgetting factor to make the model adaptive. The system developed could successfully predict the non-steady behavior of the root of the cohesive zone and hereby provide a means to design appropriate methods for controlling it. Fig. 7a illustrates the distribution of the time derivatives

of the stove temperatures, while Fig. 7b shows the predicted vertical location of the cohesive zone root in the peripheral direction of the furnace.



**Figure 7: a) Distribution of time derivative of stove temperatures.  
b) Predicted root of cohesive zone [126].**

As a final example of data-driven modeling applied to the blast furnace, statistical techniques developed by Kamo et al. [127] to predict the channeling behavior is presented. Using pressure and temperature measurements at various locations, eight indices were developed and combined into a single channeling index. Whenever the combined index value exceeded a threshold value, an alarm was raised to take preventive actions such as lowering the blast volume. The system has been applied at the Kakogawa works in Japan.

## 6. Summary

Mathematical modeling of the blast furnace has played a crucial role in adaptation of the process to the various challenges it has met. The present paper has reviewed various stages of model development. The models have been classified into first-principle models and data-driven models. Both model types serve a clear purpose. For example, lumped models can estimate top gas conditions, coke rate, productivity, or thermal and chemical demands of the furnace. 1-D comprehensive models can be a very good tools for on-line implementation purposes, whereas 2-D models help to understand the importance of radial distribution, which plays a crucial role for the gas flow and cohesive zone prediction. 3-D models can, e.g., reveal reasons for maldistribution of liquids and gases in the bosh region and explain departure from symmetry due to tuyere blanking and scaffold formation. They are also the only model type that correctly

considers the conditions in and between the raceways. Models based on discrete elements can describe complex behavior of particulate flow, e.g., in burden charging, at the interfaces between the descending layers, and in the formation of the deadman, and CFD-DEM models hold promises to describe the conditions inside the whole furnace. Data-driven models, in turn, may facilitate the optimization of furnace performance with multiple objectives, detect anomalies or estimate the root of the cohesive zone by interpretation of noisy signals at the furnace boundary. The data-driven models also make use of sensor information that is difficult to interpret by models based on idealized physical conditions.

With an increase in the computational capabilities offered by cloud computing and the advancement of GPU technology, as well as the recent developments in artificial intelligence algorithms, it is possible to develop both better data-driven models and hybrid systems which combine both model types in the future. The main challenge for the blast furnace is its use of carbon of fossil origin, which must be abandoned in the future to realize a sustainable steelmaking. However, as long as there are not technically and economically feasible alternatives, the performance of the blast furnace must be further pushed towards the theoretical limits by better and more predictive control, reduced number of disturbances and greater flexibility to operate under varying external constraints. To reach these goals, it is imperative to develop more sophisticated and intelligent mathematical models of the process.

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